STRATEGIC PLACEMENT OF HARMONIC FILTERS IN DISTRIBUTION NETWORKS

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Abstract

This paper presents an approach to identifying strategic buses for the placement of harmonic single tuned filters in a large distribution network. Information obtained from sensitivity analysis based on inherent structure theory of network (ISTN) is used in conjunction with the current injection method available in commercial software (SUPERHARM) to investigate strategic placement of harmonic filters for large distribution system. Performance of the filters under various loading scenarios is then assessed to determine their effectiveness in controlling the harmonic voltage distortion level.

Key Words

Power Quality, Harmonics, Sensitivity analysis, Passive filters

1. Introduction

There has been a constant growth in the number of electronics and power electronics devices in the customer installations. This lead to overall increased harmonic levels in distribution systems. Most of the single-phase power electronics based loads (e.g., personal computers (PCs)) are relatively low wattage devices drawing high harmonic content currents from the network. In commercial buildings in addition to fluorescent lighting, computers and their peripherals are used in large quantities and thereby may cause significant harmonic distortion at the utility substation. Higher wattage (three-phase) devices, e.g., heat pumps and central air-conditioning (HVAC) with variable speed drives also contribute to significant harmonic distortion [1].

There is generally substantial harmonic cancellation at the service entrance resulting from load diversity and/or non-linear load supply via transformers having different winding connections. The need for additional filtering however, is often necessary in modern distribution networks. Harmonic filtering is one of the solutions to prevent propagating of harmonics through the system. Single tuned filters are economical and efficient and therefore commonly used in mitigating harmonic [2]. For economic reasons, especially in large distribution network, it is desirable to minimize the number of filters be installed. Filters therefore, have to be placed at the most strategic locations of the network in order to produce the expected performance.

Harmonic filter planning methodology based on measured bus voltage harmonics at substations is described in [3]. The approach is essentially based on the characteristics of distribution systems and harmonic sources. The algorithm for optimal placement of passive filters by converting existing capacitor banks in the network with voltage distortion limits and filter component rms constraints is proposed in [4]. Both studies are targeted at small distribution systems with substation bus voltage distortions known from measurement and where harmonic filter requirement is for the purpose of reducing harmonic voltage distortion level to a predefined value.

This paper proposes an approach to identify strategic locations for the placement of single tuned harmonic filters in large distribution systems at the planning stage or when the exact harmonic voltage distortion at substations is not known. Sensitivity indices are used as the indicators of buses with potentially high voltage distortion if supplying non-linear loads. Single tuned filters are then placed at buses having highest sensitivity. The results of mitigation are verified through simulations.

2. Inherent Structure Theory Applied to Harmonic Analysis

The h harmonic order admittance matrix $\mathbf{Y}^h$ of the N-bus power system can be formulated in terms of its eigenvalues λ_i^h and eigenvectors $\mathbf{v}_i^h$ [5] as :

$$\mathbf{Y}^h = (\mathbf{V}^h) diag(\boldsymbol{\lambda}^h)(\mathbf{V}^h)^{-1} \quad (1)$$

where $\mathbf{V}^h = (\mathbf{v}_1^h, \ldots\ldots\ldots, \mathbf{v}_N^h)$ and $diag(\boldsymbol{\lambda}^h)$ is the diagonal matrix whose elements are the eigenvalues of $\mathbf{Y}^h$.
The harmonic impedance matrix is given by:

$$\mathbf{Z}^h = (\mathbf{V}^h) diag(1/\boldsymbol{\lambda}^h)(\mathbf{V}^h)^{-1} \quad (2)$$

and harmonic voltages $\mathbf{V}^h$ at each bus by:

$$\mathbf{V}^h = (\mathbf{V}^h) diag(1/\boldsymbol{\lambda}^h)(\mathbf{V}^h)^{-1}\mathbf{I}^h \quad (3)$$

where $\mathbf{I}^h$ is the harmonic current injected at each bus.

Equation (3) can be rewritten as,

$$\overline{\mathbf{V}}^h = \sum_{i=1}^{N} \frac{1}{\lambda_i^h} \mathbf{v}_i^h (\mathbf{w}_i^h)^T \overline{\mathbf{I}}^h \quad (4)$$

where $\mathbf{w}_i^h$ is the i-th left eigenvector of the matrix $\mathbf{Y}^h$.

Based on equation (4), it has been shown in [5] that the eigenvalue of minimum modulus is often associated with the scalar factor, $\{(\mathbf{w}_i^h)^T \overline{\mathbf{I}}^h\}$ of maximum modulus. It was further shown that as an approximation to compute harmonic voltages $\overline{\mathbf{V}}^h$, in most cases it is sufficient to consider only the term associated with the eigenvalue of minimum modulus, λ_k given in equation (5) below.

$$\overline{\mathbf{V}}^h \cong \frac{1}{\lambda_k^h} \mathbf{v}_k^h (\mathbf{w}_k^h)^T \overline{\mathbf{I}}^h \tag{5}$$

The sensitivity matrix $\mathbf{S}_i^h$ of the i-th eigenvalue is then

$$\mathbf{S}_i^h = \mathbf{w}_i^h (\mathbf{v}_i^h)^T = \begin{bmatrix} \frac{\partial \lambda_i^h}{\partial y_{11}^h} & . & . & . & \frac{\partial \lambda_i^h}{\partial y_{1N}^h} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ \frac{\partial \lambda_i^h}{\partial y_{N1}^h} & . & . & . & \frac{\partial \lambda_i^h}{\partial y_{NN}^h} \end{bmatrix} \tag{6}$$

Each element of the matrix $\mathbf{S}_i^h$ represents the sensitivity coefficient relating the change in the i-th eigenvalue to the element of the $\mathbf{Y}^h$ matrix.

As our interest is only in term associated with the eigenvalue of minimum modulus λ_k^h, the corresponding sensitivity matrix is

$$\mathbf{S}_k^h = \mathbf{w}_k^h (\mathbf{v}_k^h)^T = \begin{bmatrix} \frac{\partial \lambda_k^h}{\partial y_{11}^h} & . & . & . & \frac{\partial \lambda_k^h}{\partial y_{1N}^h} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ \frac{\partial \lambda_k^h}{\partial y_{N1}^h} & . & . & . & \frac{\partial \lambda_k^h}{\partial y_{NN}^h} \end{bmatrix} \tag{7}$$

3. Model Of The Distribution Network

Fig. 1A in the Appendix shows the distribution network used for the analysis and harmonic simulation. The network consists of 25-33kV buses, 233 -11kV buses and 4-3.3kV buses. The network is fed from the 400 kV, 275 kV and 132kV grid system through step down transformers. A major portion of the 33kV and 11kV network is of meshed configuration. Overhead lines and underground cables are modeled by π equivalent circuit, transformers by their short circuit and magnetizing impedances and linear loads by frequency dependent resistance in parallel with inductance [6].

4. Sensitivity Analysis

The sensitivity of each bus of the distribution network to the addition of a shunt filter is found based on the k-th eigenvalue sensitivity matrix given by (7). The sensitivity is indicated by the values of $\left|\partial \lambda_k^h / \partial y_{ii}^h\right|$ which are the diagonal elements of the k-th sensitivity matrix, $\mathbf{S}_k^h$. The largest value corresponds to the most sensitive bus. A normalized index to rank the buses of the network was found by taking the ratio of each diagonal element $\left|\partial \lambda_k^h / \partial y_{ii}^h\right|$ over the largest diagonal element. Consequently, a cumulative sensitivity index, CSI is obtain as,

$$\mathrm{CSI}_i = \frac{\sum_{h=1}^{M} \partial \lambda_k^h / \partial y_{ii}^h}{\max\left\{\sum_{h=1}^{M} \partial \lambda_k^h / \partial y_{ii}^h\right\}} \tag{8}$$

where $i=1, 2, 3, \ldots\ldots\ldots, N$ is the respective bus number, and M is the maximum harmonic order.

Fig. 2 shows a graphical representation of the cumulative sensitivity index CSI of all the buses in the network.

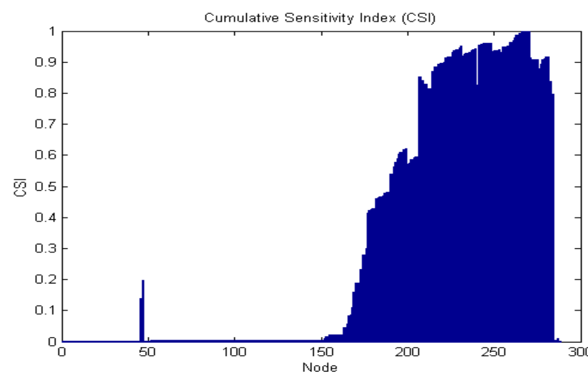

Fig. 2 – Cumulative sensitivity index indicating the relative level of sensitivity of each bus.

It can be observed from Fig. 2 that bus 270 has the highest rank and thus is the most sensitive bus of the network. This is general characteristic of distribution feeder ends (see Fig. 1) that normally experience the highest harmonic voltage distortion [3].

All buses in the network are grouped in four sensitivity groups based on their CSI.

1. Highly sensitive (HS) buses, 0.6<CSI<1.0 (E.g., buses 207 - 284 (11kV) group ‘A’ in Fig. 1A)
2. Sensitive buses (S), 0.4<CSI<0.6 (E.g. buses 177 - 206 (11kV) group ‘B’ in Fig. 1A)
3. Less sensitive buses (LS), 0.1<CSI<0.4 (E.g., buses 168-176 (11kV), 46&47 (33kV) group ‘C’ in Fig. 1A)
4. Non-sensitive buses (NS), CSI <0.1 (E.g., buses 27-45 (33kV), 52-167 (11kV) and 285-288 (3.3kV) group ‘D’ in Fig. 1A)

5. Frequency Scan Analysis

Frequency scan analysis is used to determine frequency response of a power system [7]. This is done with a current or voltage source of $1\angle 0°$ p.u injected at one or more buses of the power system for a range of frequencies of interest. In this study, frequency scan is done using SUPERHARM to verify the results of bus sensitivity analysis described in Section 4. A 1 p.u multiple frequency harmonic current source, $\overline{I}^h$ was injected at the bus of interest to determine the magnitude of the corresponding harmonic voltages, $|\overline{V}^h|$ as shown in Fig. 3, where $\overline{Z}_i^h$ is the harmonic driving point impedance at bus *i*. The results of the 5th and 7th harmonic for 5 selected buses representing areas of high, medium and low sensitivity are shown in Table 1. Only the 5th and 7th harmonics are selected in this case as these harmonics are expected to be the most dominant on the 11kV system [8]. Since most 11/0.4 kV distribution transformers are delta-wye connected, triplen harmonics are ignored [9].

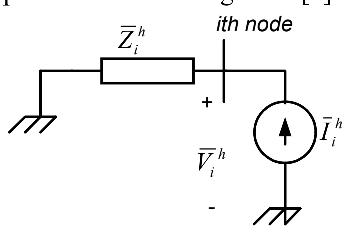

Fig. 3 – Current injection method

The results in Table 1 show a clear correlation between the bus sensitivity index and harmonic driving impedance (indicated by the magnitude of the harmonic voltage). It can be seen that the higher the sensitivity index the higher is it's harmonic driving impedance.

Table 1 - Bus sensitivity obtained from sensitivity analysis and harmonic voltage from current injection method.

	5TH Harmonic		7TH Harmonic	
Buses, *I*	Sensitivity Index	$\lvert\overline{V}^5\rvert$	Sensitivity Index	$\lvert\overline{V}^7\rvert$
270	1.0	25.0031	1.0	35.6210
251	0.9142	13.6151	0.9192	19.7119
222	0.8847	11.1761	0.8931	16.2613
208	0.7082	8.0136	0.7342	11.8982
172	0.0028	6.9822	0.0013	9.2188
136	0.0011	1.1479	0.0008	0.5959

Frequency scan with current injection of $1\angle 0°$ p.u simultaneously applied at 3 selected buses (29(NS), 168(LS), 264(HS)) is performed to determine the frequency response of the system. The harmonic voltage profile shown in Fig. 4 is also indicative of the sensitivity of respective buses and it is in agreement with Fig. 2. The frequency responses of seven selected buses having different sensitivities (34, 39, 67, 47, 161, 208, 281) are shown in Fig. 5 and 6. It can be seen that the LS buses (Fig. 5) have two resonant frequencies (15th and 5th harmonic) while HS buses (curves, 47, 208 and 281 in Fig. 6) have

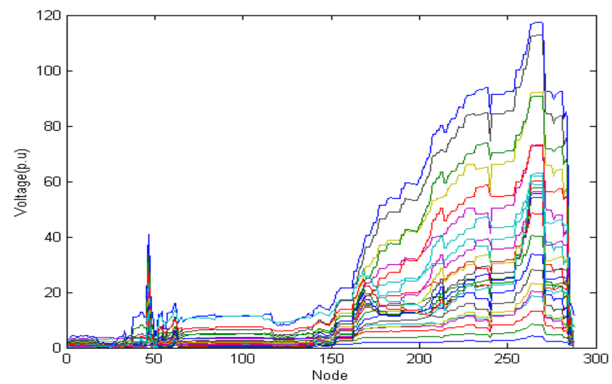

Fig. 4 – Harmonic voltage profile resulting from$1\angle 0°$ p.u current injection at buses 29, 168 & 264.

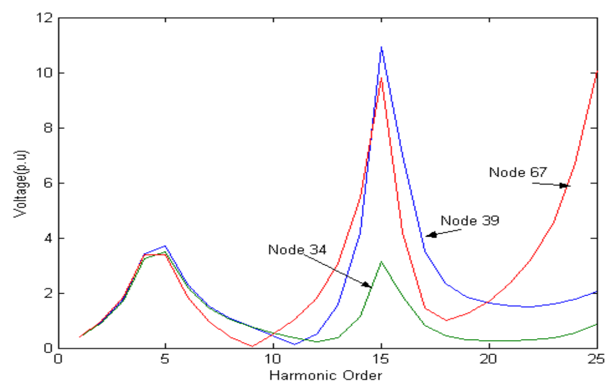

Fig. 5 – Frequency response of characteristic LS buses with $1\angle 0°$ p.u current injection at buses 29, 168 & 264.

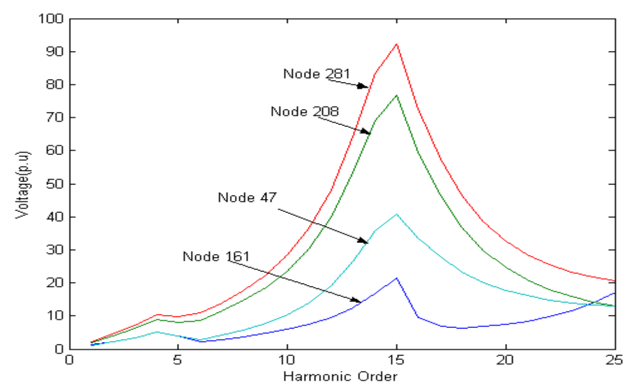

Fig. 6 – Frequency response of characteristic LS buses with $1\angle 0°$ p.u current injection at buses 29, 168 & 264.

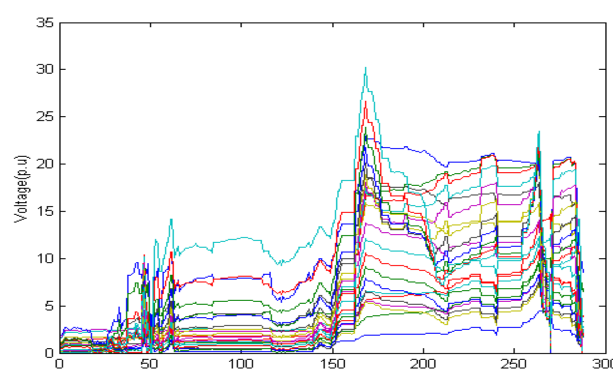

Fig. 7 - Harmonic voltage profile resulting from$1\angle 0°$ p.u current injection at buses 29, 168 & 264, 15th harmonic filter at bus 270

only one dominant resonant frequency (15th harmonic). This information will be used to identify strategic buses for placement and tuning of filters in Section 7.

6. Tuning of Harmonic Filters

The single tuned filters only are considered in this study as they are simple to design and the least expensive. It is assumed that the power factor correction is not required and that the capacitor size can be selected without paying attention to that constrain.

Consider a series, single tuned filter with the following parameters: X_C - capacitor fundamental frequency reactance, X_L - reactor fundamental frequency reactance and R - series resistor. At the tuned (critical) frequency ω_n,

$$\frac{1}{n\omega_0 C} = n\omega_0 L \qquad (9)$$

and,

$$n\omega_0 = \omega_n = \frac{1}{\sqrt{LC}} \qquad (10)$$

where ω_0 is the power system fundamental frequency, ω_n is the tuned frequency of the filter and n is the harmonic order of the tuned frequency. For relatively small values of R, the fundamental frequency reactive power is given by,

$$Q_f = \frac{|V|^2}{(X_L - X_C)} \qquad (11)$$

since $X_L = X_C / n^2$, the reactive power of the series tuned filter is given by

$$Q_f = \frac{|V|^2}{\left(\frac{X_C}{n^2} - X_C\right)} = \frac{|V|^2}{X_C} \cdot \frac{n^2}{(1 - n^2)} \qquad (12)$$

where $|V|$ is the magnitude of the fundamental voltage at the bus where the filter is connected. From (12), the value of X_C is selected based on the reactive power Q_f to be supplied by the filter if power correction is to be considered. Reactance X_L is then selected as $X_L = X_C / n^2$. Finally, R is determined by the filter quality factor Q which is defined as $Q = nX_L / R$ [10].

7. Placement of Harmonic Filters

Sensitivity analysis performed in Section 4 identified the relative sensitivity of each bus of the network. Buses 207-284 being the most sensitive are therefore, candidate buses for the placement of shunt filter. However, in cases when the location of harmonic source is known, the most effective bus to place a filter would be at the location of the harmonic source.

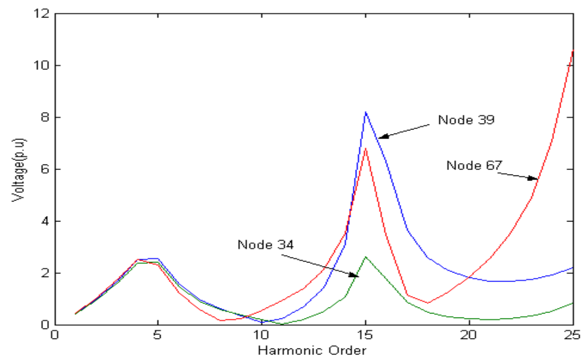

Fig. 8 - Frequency response of characteristic LS buses with 1∠0° p.u current injection at buses 29, 168 & 264, 15th harmonic filter at bus 270

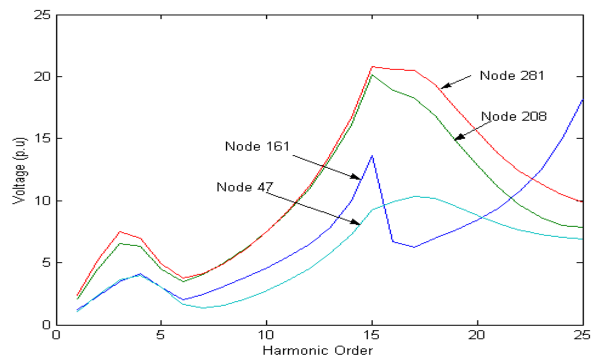

Fig. 9 - Frequency response of characteristic HS buses with 1∠0° p.u current injection at buses 29, 168 & 264, 15th harmonic filter at bus 270

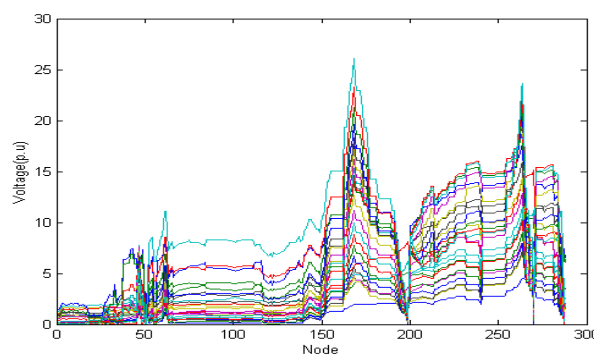

Fig. 10 - Harmonic voltage profile resulting from1∠0° p.u current injection at buses 29, 168 & 264, 15th harmonic filter at buses 270 & 199 and 5th harmonic filter at bus 101

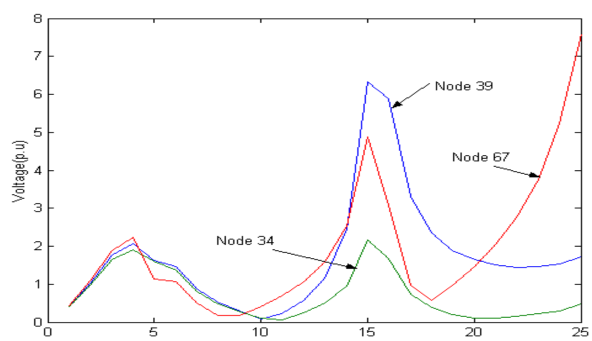

Fig. 11 - Frequency response of characteristic LS buses with 1∠0° p.u current injection at buses 29, 168 & 264, 15th harmonic filter at buses 270 and 199 and 5th harmonic filter at bus 101

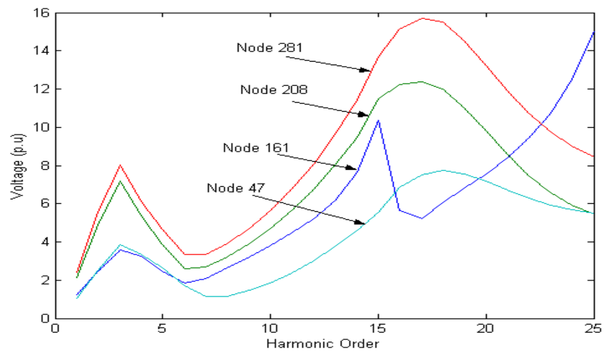

Fig. 12 - Frequency response of characteristic LS buses with 1∠0° p.u current injection at buses 29, 168 & 264, 15th harmonic filter at buses 270 and 199 and 5th harmonic filter at bus 101

In this analysis it is assumed that the harmonic sources (non-linear loads) are spread throughout the network at all buses and that the harmonic current level at each bus varies according to the loading condition. For simplicity, it is assumed that the harmonic current level at every bus is about the same. For the case study considered in this analysis (current injection at buses 29(NS), 168(LS) and 264(HS)), the most effective bus to place the filter would be the most sensitive (*CSI*=1.0) one, i.e., bus 270. Based on the frequency response curves, a 15th harmonic filter is tuned and placed at bus 270. The simulation results with this filter in place are shown in Fig. 7-9. Significant improvement of the network harmonic performance in this case compared to the results shown in Figures 4-6 can be seen.

With the filter at bus 270, a second set of CSI is calculated to identify new most sensitive bus of the system. The second 15th harmonic filter is tuned and placed at bus 199. This further improved frequency response of the network.

The above process is repeated with filters at buses 270 and 199. The most sensitive bus for filter placement in this case happen to be bus 101. The 5th harmonic filter is tuned this time as the 15th harmonic filter resulted in a very small improvement compared to the previous case. The simulation results with three filters are shown in Fig. 10-12.

The frequency responses of the most sensitive bus (270) without and with different number of filters installed are shown in Fig. 13. The effectiveness of the first installed filter (at bus 270) is obvious while the other filters do not influence significantly harmonic voltages at this bus.

8. Robustness of the Solution

With appropriately tuned harmonic filters placed at buses 270, 199 and 101, the performance of the distribution network is assessed using five test cases four of which are summarized in Tables 2–5 with corresponding harmonic spectrums in Tables 6-8. The results of simulations are shown in Figures 14 -18. These figures show that the voltage total harmonic distortion (THD_V) depends on both, the location of harmonic sources as well as on sensitivity of the buses.

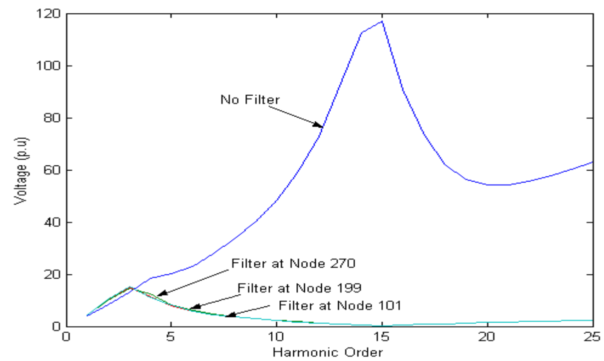

Fig. 13 – Frequency response of the most sensitive bus 270 without and with different filters installed

In Fig. 15 for example, the THD_V is almost leveled throughout the network when there is a high concentration of harmonic sources (non-linear loads) in the area of lowest sensitivity. Buses with higher sensitivity are “compensated for” by the flow of lower harmonic currents. Reduction in THD_V is almost the same at every bus as a result of the connection of filters in all cases except in the Case III. In this case, there are a few buses (160-170) which show only a very small reduction in THD_V following the installation of filters.

Table 2. Case I – Evenly spread non-linear loads

Type of nonlinear load	Buses
Industrial	29, 110, 253
Commercial	52, 90, 147, 185, 217
Residential	118, 172, 195

Table 3. Case II – High concentration of non-linear loads in the NS area

Type of nonlinear load	Buses
Industrial	29, 86, 110
Commercial	53, 78, 88, 114, 129
Residential	52, 72, 135

Table 4. Case III – High concentration of non-linear loads in S to NS area

Type of nonlinear load	Buses
Industrial	45, 159, 189
Commercial	54, 100, 115, 155, 173
Residential	140, 162, 188

Table 5. Case IV - High concentration of non-linear loads in the HS area.

Type of nonlinear load	Buses
Industrial	47, 236, 253
Commercial	193, 211, 247,283
Residential	200, 227, 238

Case V - Evenly spread non-linear loads with 50% reduction in linear load.

Table 6 - Non linear loads of industrial customers

Harmonic	ASD [7]		rectifiers[11]	
	mag	angle	mag	angle
1	1	-1.5	1	-10
3	0.542	0.7	0	0
5	0.152	110.8	0.67	120
7	0.069	151.9	0.429	-90
9	0.043	-95.0	0	0
11	0.036	-13.9	0.095	-120
13	0.029	95.2	0.074	60
15	0.025	-182.7	0	0

Table 7 - Non linear loads of commercial customers

Harmonic	fluorescent lights [7]		computers [12]	
	mag	angle	mag	angle
1	1	-41.2	1	0
3	0.2	273.4	0.81	0
5	0.107	339.0	0.53	0
7	0.021	137.7	0.25	0
9	0.014	263.2	0.09	0
11	0.009	39.8	0.05	0
13	0.006	182.4	0.04	0
15	0.005	287.0	0.03	0

Table 8 - Non linear loads of residential customers

Harmonic	television sets [13]		Others [7]	
	mag	angle	mag	angle
1	1	0	1	-7.0
3	0.798	-173.0	0.007	-105.8
5	0.492	12.0	0.6	-57
7	0.20	-159.0	0.36	87
9	0.04	81.0	0	0
11	0.055	-13.0	0.17	-21
13	0.031	175.0	0.13	114
15	0.008	-138.0	0	0

9. Conclusions

The paper presented a simple and effective methodology for identifying sensitive buses in distribution network with respect to harmonic distortion without having to perform any measurements. The method is particularly useful at the network planning stage where exact harmonic voltage/current distortion is not known

The results are compared using standard current injection method and a good agreement is observed. Based on sensitivity information and frequency scan, harmonic filters are strategically located throughout the network and tuned to improve network harmonic performance.

The robustness of the solution is confirmed in tests with different non-linear loads, different load composition and different locations of non-linear loads.

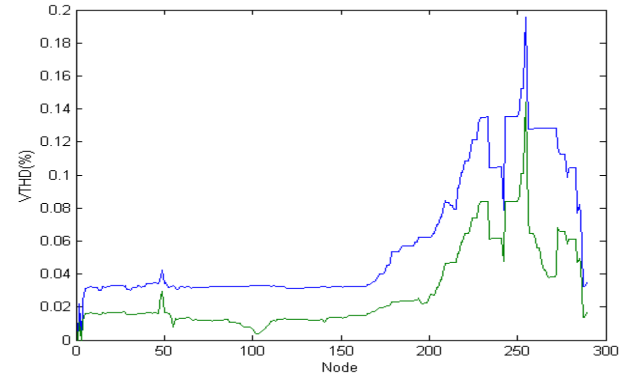

Fig. 14 – Case I, Voltage THD with and without filters at bus 270, 199 &101

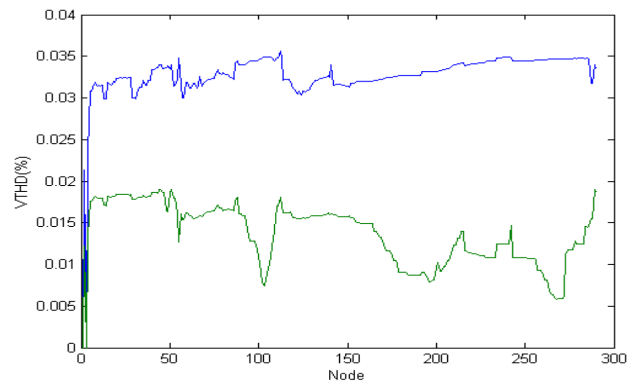

Fig. 15 – Case II, Voltage THD with and without filters at bus 270, 199 &101

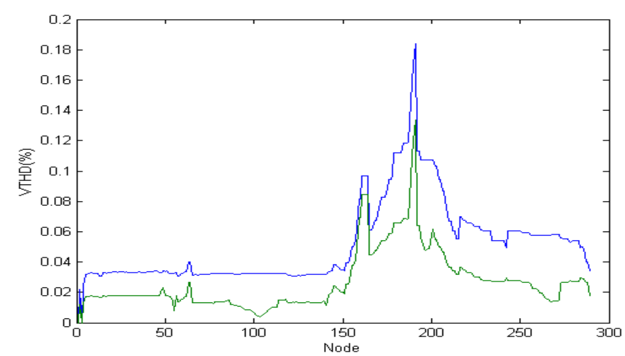

Fig. 16 – Case III , Voltage THD With and Without filters at Bus 270, 199 &101

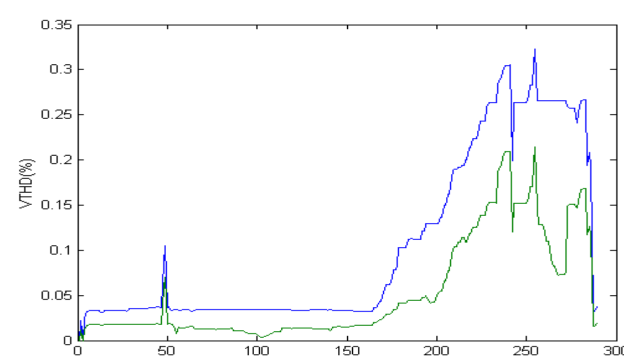

Fig 17 – Case IV, Voltage THD with and without filters at bus 270, 199 &101

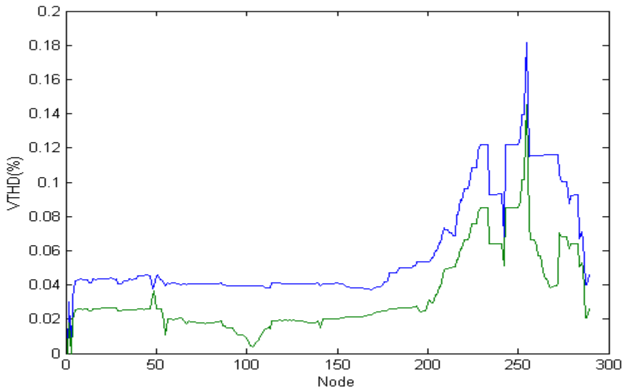

Fig. 18 - Case V, Voltage THD with and without filters at bus 270, 199 &101

10. Acknowledgement

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Appendix

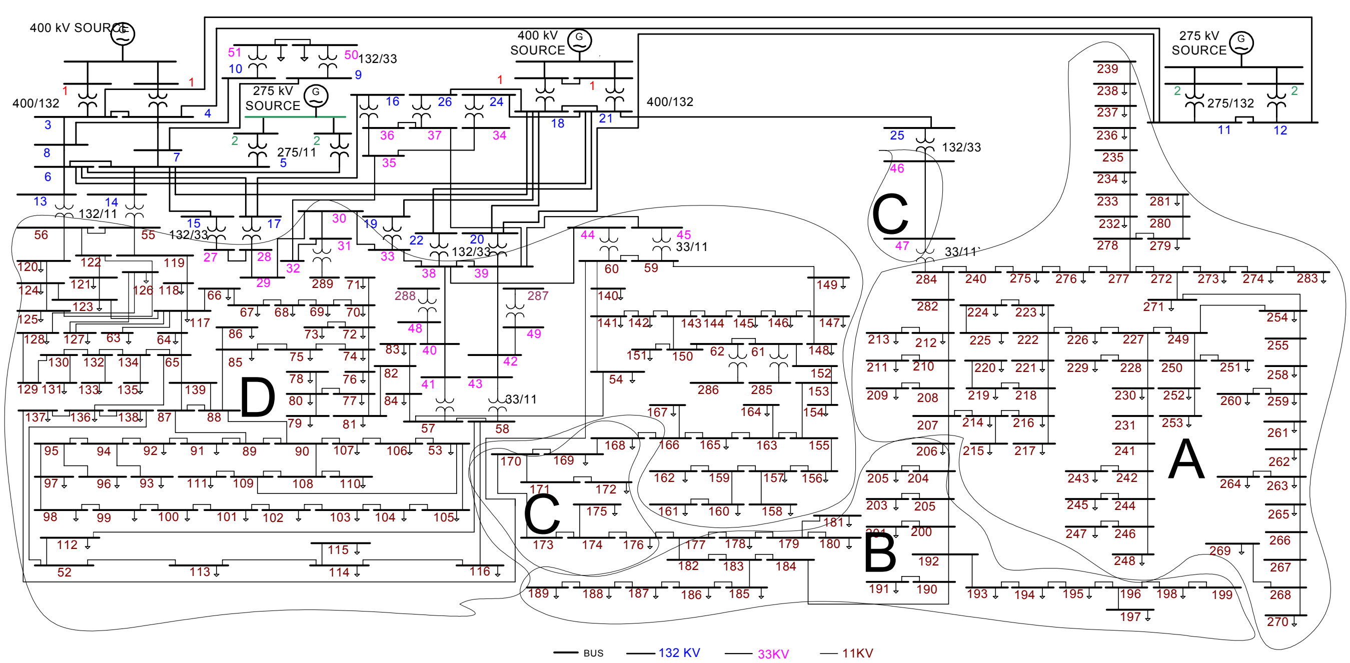

Fig. 1A – Single line diagram of Generic Distribution Network

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Number of Pages: 7
Number of Words: 4,269 (approx.)
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