THE SIMULATION OF LOW VOLTAGE DISTRIBUTION POWER SYSTEM BEHAVIOUR UNDER NON-SINUSOIDAL CONDITIONS - CASE STUDY

Andrzej BACHRY, Zbigniew STYCZYNSKI
Chair Electrical Power Networks and Renewable Energy Sources,
Institute of Electric Power Systems,
Otto-von-Guericke-University of Magdeburg,
D-39016 Magdeburg, Germany

Abstract – The paper presents the study of the distribution system behaviour under distorted conditions using SuperHarm® software packet. The modelling of a distribution power system for harmonic penetration is presented and discussed with special emphasis onto definitions of power. Some remarks concerning SuperHarm® procedures used during investigations are presented. Examples and discussion is provided.

1. INTRODUCTION

Semiconductor devices used in the distribution system introduce harmonic current in power system. This current is propagated throughout the power network and originate many disturbances, which can affect secure operation of the network, for instance may cause defective erection of protection devices. Moreover the standards of supply parameters required by customers can not be in all cases guaranteed [1]. The resulting enhance of the voltage waveform distortion turn out with the appearance of harmonic frequencies in the network voltage supplying other devices.

2. MAIN SOURCES OF HARMONICS

Harmonic disturbances observed in the network have their origins in two main categories. The first one includes devices with strong non-linear u-i relationship; the second group covers a wide range of mentioned electronic switching equipment [2].

The first category refers mainly to electric metal-melting arc furnaces, transformers, fluorescent, and other gas discharging lighting [3]. The modeling difficulty is connected here with the variability of the current harmonic injections to be used in each particular study, which should be based on extensive experimental information obtained from measurements in similar existing installations [4].

In the second group there is a large number of adjustable speed motor drives (ASDs), battery charges, electronic ballasts and other rectifier/inverter applications. These appliances present also a simulation problem, in some cases statistical information of their content in the load mix is provided, sometimes data are obtained from extensive measurements or theoretical analysis.

Assuming, all this equipment needs non-sinusoidal current from supply and this distorted current flows through the network. In other words, each of this apparatus is a source of harmonics that are spread over the system. The problem is in the next few years the percentage of electronic controlled load in middle and low voltage networks will be about 70. That big amount of this type of load will cause much more difficulties than today does [2].

3. HARMONIC STANDARDS AND DEFINITIONS OF POWER

In view of the widespread use of power electronic equipment connected to utility systems, various national and international agencies have proposed limits on harmonic current injection into the system by this equipment [5, 6]. There are several measures, which are the main definitions for most common indication of harmonics presence like *Total Harmonic Distortion* (*THD*), which can be obtained for either voltage or current, and *Total Demand Distortion* (*TDD*) used during calculation of current distortion at the point of common coupling between customer and utility.

There are also calculated other indices, such as *Telephone Influence Factor* (*TIF*) and *K – factor*. The first one is used to measure telephone interference, the second to describe the impact of harmonics on losses and is often used in de-rating of equipment, e.g. transformers [5].

Power properties of circuits with non-sinusoidal waveforms are nowadays strongly discussed [7, 8, 9]. There is still no consensus in the interpretation and definition of non-active powers in circuits with distorted and/or unbalanced voltages and currents. Therefore three main methods of power resolution were chosen namely that of Budeanu, Czarnecki, and the IEEE Working Group and compared for power calculations at main bus in low voltage distribution system.

Main aspects of Budeanu power definitions

Representing voltage and current as complex, root-mean-square scaled, time dependent quantities [10]:

$$u(t) = \sqrt{2}\,\mathrm{Re}\left\{\sum_n \mathbf{u_n(t)}\right\} \text{ and } i(t) = \sqrt{2}\,\mathrm{Re}\left\{\sum_n \mathbf{i_n(t)}\right\}, \tag{1}$$

where:

$$\mathbf{u_n(t)} = U_n \cdot e^{j(n\omega t + \alpha_n)} \text{ and } \mathbf{i_n(t)} = I_n \cdot e^{j(n\omega t + \beta_n)} \tag{2}$$

are the constituent n-th order time-dependent complex, root-mean-square scaled quantities over the harmonic range, α_n and β_n are the phase angles of the voltage and current, respectively. The total time-dependent real power (in all harmonic orders) can be defined as:

$$p(t) = \sum_n \sqrt{2}\,\mathrm{Re}\{\mathbf{u_n(t)}\}\cdot\sqrt{2}\,\mathrm{Re}\{\mathbf{i_n(t)}\}. \tag{3}$$

When substituting in (3) for $\boldsymbol{u_n(t)}$ and $\boldsymbol{i_n(t)}$ from (2) and after manipulation we obtain:

$$P = \sum_n U_n \cdot I_n \cdot \cos\varphi_n , \tag{4}$$

which is an average power or *active power* with $\varphi_n = \alpha_n - \beta_n$. According to Budeanu the instantaneous imaginary power is defined as:

$$q(t) = \sum_n \sqrt{2}\,\mathrm{Re}\{\mathbf{u_n(t)}\}\cdot\sqrt{2}\,\mathrm{Im}\{\mathbf{i_n(t)}^*\}, \tag{5}$$

where the asterisk denotes a vector of the conjugate values. Then, treated analogously, (5) leads to the average of this expression, which is termed by Budeanu as the *reactive power*:

$$Q = \sum_n U_n \cdot I_n \cdot \sin\varphi_n . \tag{6}$$

Since cosine-function in (4) forces *P* to be the arithmetic sum of the real powers in all harmonic orders, the sine-function in (6) may result in a zero value for *Q*, while the constituent harmonic orders have non-zero values.

Czarnecki power resolution

To avoid misleading interpretation of non-active powers under distorted and/or unbalanced conditions Czarnecki has proposed another concept of power resolution in circuits with non-sinusoidal waveforms [7].

This theory bases on the decomposition of currents into orthogonal components. Non-active powers are then defined as the product of the rms value of voltage by the rms value of current components. Since the currents, active and reactive powers do not change with the change of the voltage reference in three-phase circuits, an increase in the apparent power with the load unbalance should be explained in terms of the current rms value [11].

Decomposition bases on the partitioning of the set of N harmonic orders into two subsets N_A and N_B, namely:

$$\begin{aligned} &\text{if } P_n \geq 0 \text{ then } n \in N_A \\ &\text{if } P_n < 0 \text{ then } n \in N_B \end{aligned}, \tag{7}$$

where P_n is the active power transmitted from the source to the load at the n-order harmonic frequency amounts to:

$$P_n = \operatorname{Re}\{\mathbf{S_n}\} = \operatorname{Re}\{\mathbf{U_n}^T \cdot \mathbf{I_n}^*\}, \tag{8}$$

in which the vectors of voltages and currents are calculated as complex root mean square values for each harmonic frequency. In three-phase circuits these quantities can be obtained for line-to ground voltages and line currents:

$$\mathbf{U_n} = [U_{Rn}, U_{Sn}, U_{Tn}]^T,\ \mathbf{I_n} = [I_{Rn}, I_{Sn}, I_{Tn}]^T. \tag{9}$$

An explanation for this decomposition lies in the direction of the energy flow. When the load is passive, linear, and time-invariant, then each of harmonic active power P_n can not be negative. However, if any of these conditions is not fulfilled, the harmonic currents may be generated in the load, so that the energy at that frequency may be transmitted back to the source (P_n<0). This decomposition also enables to decompose the current observed at the bus into two mutually orthogonal components [11], and their rms values fulfil the relationship:

$$\|\mathbf{i}\|^2 = \|\mathbf{i_A}\|^2 + \|\mathbf{i_B}\|^2. \tag{10}$$

Similar relationship for the voltage yields to the statement describing the *apparent power S*, which was first defined by Buchholz and then extended by Czarnecki:

$$S^2 = \|\mathbf{u}\|^2 \cdot \|\mathbf{i}\|^2 = S_A{}^2 + S_B{}^2 + S_F{}^2 \tag{11}$$

with

$$S_A = \|\mathbf{u_A}\| \cdot \|\mathbf{i_A}\|,\ S_B = \|\mathbf{u_B}\| \cdot \|\mathbf{i_B}\|,\ S_F = \sqrt{\|\mathbf{u_A}\|^2 \cdot \|\mathbf{i_B}\|^2 + \|\mathbf{u_B}\|^2 \cdot \|\mathbf{i_A}\|^2}. \tag{12}$$

The last quantity in (12) occurs also in single-phase circuits under distorted conditions and is called *forced apparent power*. From (11) can be observed that the presence of bi-directional transmission of active power reduces the transmitted useful power P, while increasing the apparent power S due to an increase in the voltage and current rms values. Continuing the decomposition of the current i_A Czarnecki has separated next four orthogonal components bounded to separate power phenomena in such circuits [11]. The details can be found in several papers of Czarnecki.

IEEE Working Group definitions for powers

The concept presented in [12], which is also similar in [13], bases on the main assumption that the object of transmission is to deliver as much of the power as possible through 50Hz positive sequence component to the consumer. Therefore it makes sense to separate the fundamental and the harmonic components from each other. Using the rms values of the voltage and current:

$$U = \sqrt{\sum_n U_n{}^2},\ \ I = \sqrt{\sum_n I_n{}^2} \tag{13}$$

the harmonic rms-components can be separated into their fundamental and harmonic components:

$$U^2 = U_1^2 + U_H^2,\ I^2 = I_1^2 + I_H^2, \tag{14}$$

with:

$$U_H^2 = \sum_{n \neq 1} U_n^2,\ I_H^2 = \sum_{n \neq 1} I_n^2, \tag{15}$$

respectively. From (14) the *apparent power* is defined:

$$S^2 = (U \cdot I)^2 = S_1^2 + S_N^2. \tag{16}$$

And is divided into two main components, where S_1 is the *fundamental apparent power*, which is in turn resolved into the *fundamental active power* P_1 and *fundamental reactive power* Q_1 according to well-known equation used under pure 50Hz sinusoidal conditions. The second component in (16) is named *non-fundamental apparent power* S_N, and consists of three components:

$$S_N^2 = (U_1 \cdot I_H)^2 + (U_H \cdot I_1)^2 + (U_H \cdot I_H)^2. \tag{17}$$

Because of the fact, that the first component is the product of fundamental rms voltage and harmonic current, it is named *current distortion power*, and usually this is a dominant term in (17). The second term, thinking in similar way, is named *voltage distortion power*, and it is a reflection of the voltage distortion at the bus. The third term is called *harmonic apparent power* S_H. Division (17) by S_1 yields to *normalized non-fundamental distortion power*, which can be drawn using the term defined in [5]:

$$(S_N / S_1)^2 = (THDi)^2 + (THDu)^2 + (THDi \cdot THDu)^2. \tag{18}$$

For three phase situations the IEEE Working group introduces a term *system apparent power* S_e or equivalent apparent power:

$$S_e = 3 \cdot U_e \cdot I_e, \tag{19}$$

where

$$U_e = \sqrt{\frac{U_a^2 + U_b^2 + U_c^2}{3}},\ I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}}. \tag{20}$$

Voltages in (20) are line-to-neutral rms voltages calculated according to (13). Using the same approach as was used for the single-phase case, analogously named values for powers can be obtained.

4. MODELING OF DISTRIBUTION NETWORKS FOR ANALYSIS OF HARMONICS

The most common technique used for harmonic analysis is the frequency scan, in which frequency response at particular node or bus is calculated [3]. Typically a one per unit sinusoidal current is injected into the bus of interest and the voltage response is calculated using discrete frequency steps throughout the range of interest, according to equation:

$$\mathbf{Y}_n \cdot \mathbf{U}_n = \mathbf{I}_n, \tag{21}$$

where $\boldsymbol{I_n}$ is the known current vector and $\boldsymbol{U_n}$ is the nodal voltage vector to be solved.

When more data are available the frequency scan can be extended to determine additional harmonic distortion information. The one per unit harmonic current is then replaced by a specific harmonic current according to data obtained either from measurement or from literature. This approach has been expanded to cases with multiple sources of harmonics in a harmonic analysis program - SuperHarm® developed by Electrotek Concepts in USA, Tennessee [14]. The results are the harmonic voltages created by the harmonic-producing equipment.

In other words: the most common way of harmonic analysis is performed using steady state, linear circuit solution techniques. Harmonic sources, which are non-linear elements, are generally considered to be injection sources into the network models. The problem of distribution system modelling becomes nowadays an area of interest to many scientists. Significant areas of concern here include network equivalents in the view of desired accuracy together with feasibility, load modelling in search of aggregate procedures, and solution algorithms that cover a wide range of problems [2]. In this paper models available in SuperHarm® were used.

5. DISTRIBUTION SYSTEM - CASE STUDY

The real low voltage distribution system (Fig.1) consisting of step-down transformer, power factor correction capacitance, linear and power electronic load was modelled.

The electrical data of the system components were taken from available devices and equipment descriptions; length of cables was acquired approximately. All capacitances and the inductive coupling between phases in all cables were neglected. The source system was modelled as a purely sinusoidal source with an impedance derived from the short-circuit data. The background load mix was chosen according to possessed data and measurements of the voltage at the buses to achieve *THDu* about 2.5% at cubicle bus. Typical harmonic producing loads were connected as illustrated in Tab.1. After that two type of non-linear loads were connected to the system. The shapes of the current waves for both non-linear loads were taken from measurements. The appropriate power density and harmonic contents was calculated, saved in a SuperHarm® library file and used in simulations on conditions, that during power regulation (in the program) the shape of the current curve is constant. This can be seen for example, as a presence of many devices of the same type, which all are working simultaneously. And, at first, was assumed there is no phase difference between these devices. From this point of view it can be seen as the worst case of the conditions of system operation.

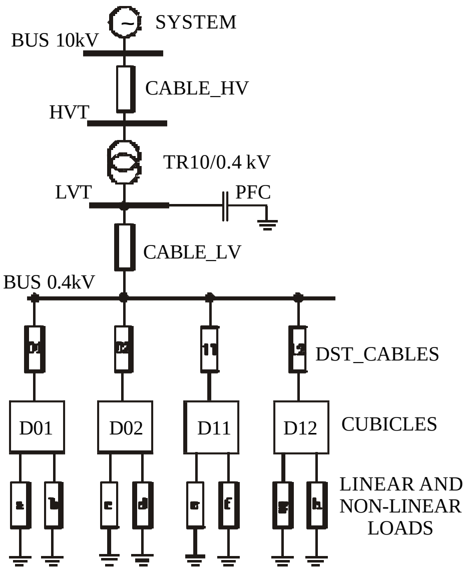

Fig.1. Distribution system.

Table 1. Case study data.

Cases \ Load	Description							
	a	b	c	d	e	f	g	h
Case 1 (background)	SMPS, 10kVA, dpf=0.97, THDi=77.3%	linear, 40kVA, pf=0.88	fluor.light, 8kVA, dpf=0.97 THDi=21.7%	free	SMPS, 10kVA, dpf=0.97, THDi=77.3%	linear., pf=0.91 a,c: 20kVA, b: 40kVA	fluor.light, 8kVA, dpf=0.97 THDi=21.7%	linear, 40kVA, pf=0.88
Case 2	as Case 1 plus as a *load d* was connected 3-phase non-linear load type A (THDi=53.72%),							
Case 3	as Case 1 plus as a *load d* was connected 3-phase non-linear load type B (THDi=115.29%),							

Case 1 describes the system operation under normal (background) conditions. Waveform of voltage and current at the *Bus0.4kV* are shown in Fig. 2. These conditions were chosen to be the starting point for the next two cases.

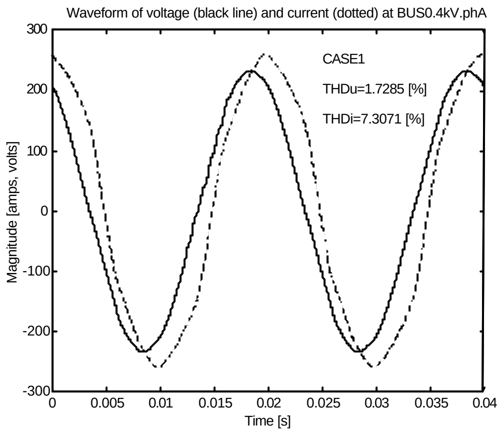

Fig.2. Waveform of voltage (black line) and current (dotted line) at *Bus0.4kV* in phase *A*, Case 1, S_L=0kVA.

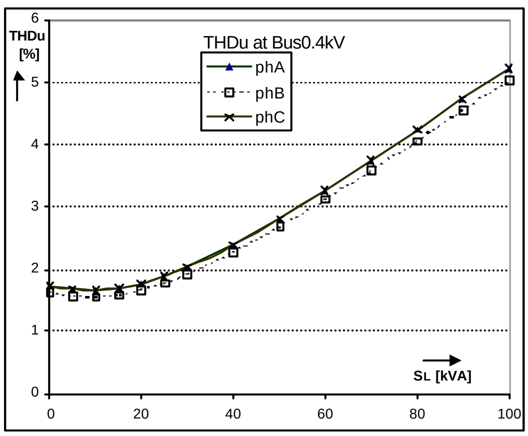

Fig. 3. *THDu* at *Bus0.4kV* versus non-linear load power S_L (Case2).

In case 2 a non-linear, harmonic producing load with *THDi*=53.72% was connected at load position *d*. The apparent power of this type of load (S_L) was regulated from 0 to 100kVA. Total Harmonic Distortion at the *Bus0.4kV* for three phases as a function of the load power S_L is showed in Fig. 3.

Case 3 was performed with similar conditions as in Case 2, except at position *d* another type of non-linear load with about two times higher *THDi*=115.29% was connected.

Both non-linear loads were modelled as three-phase loads using *NONLINEAR* load model available in SuperHarm® device library and not *ISOURCE* model, because the phase difference between load current and supply voltage was not available.

The *Total Harmonic Distortion* for the voltage in phase *B* at *Bus0.4kV* for two types of non-linear load is compared in Fig. 4. Calculated values of voltage harmonic distortion over the distribution buses for the first type of non-linear load are shown in Fig. 5.

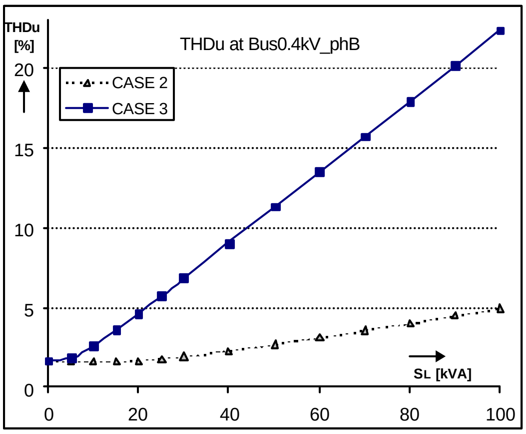

Fig. 4. *THDu* in phase *B* at *Bus0.4kV* for first (Case2) and second (Case3) type of non-linear load.

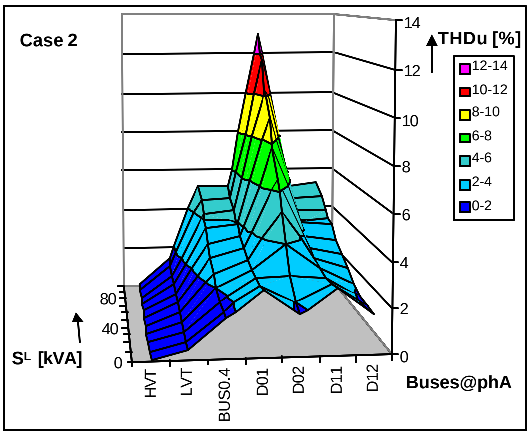

Fig. 5. *THDu* in phase *A* at buses calculated for the first type of non-linear load.

Using the data transmitted from the output processor TOP 2000 to the special prepared calculation sheet in Microsoft Excel® several power quantities were obtained at the main bus according to the described three power theories. Some values are shown in Tab. 2.

Table 2. Power calculations at *Bus0.4kV*.

Power [kVA] at *Bus0.4kV*		S_L = 0kVA	S_L = 10kVA		S_L = 50kVA		S_L = 100kVA	
		Case 1	Case 2	Case 3	Case 2	Case 3	Case 2	Case 3
BUDEANU	ACTIVE	181.713	190.625	190.612	224.905	224.556	264.741	263.383
	REACTIVE	86.643	89.616	89.451	101.243	96.453	115.160	96.447
	DISTORTION	15.205	12.369	17.203	21.913	66.633	48.014	139.151
CZARNECKI	APPARENT	204.640	212.799	213.156	249.500	258.030	296.852	330.193
	ACTIVE	181.727	190.636	190.636	224.937	224.937	264.871	264.873
	REACTIVE	86.853	89.773	89.773	101.685	101.682	116.983	116.970
IEEE	Syst.APP.	203.622	212.658	212.910	248.988	254.626	293.781	314.102
	Fund.APP.	203.168	212.382	212.382	248.243	248.243	290.704	290.704
	Fund. ACTIVE	183.363	192.201	192.201	226.265	226.265	265.978	265.978
	Fund.REACTIVE	87.494	90.359	90.359	102.121	102.121	117.322	117.322

Changes of Reactive Power calculated using Budeanu, Czarnecki and IEEE approach versus the non-linear load power S_L are presented in Fig. 6.

The extreme situation at the main bus, when the second type of non-linear load is working with 100kVA is presented in Fig. 7, where we see high-distorted waveforms of current and voltage

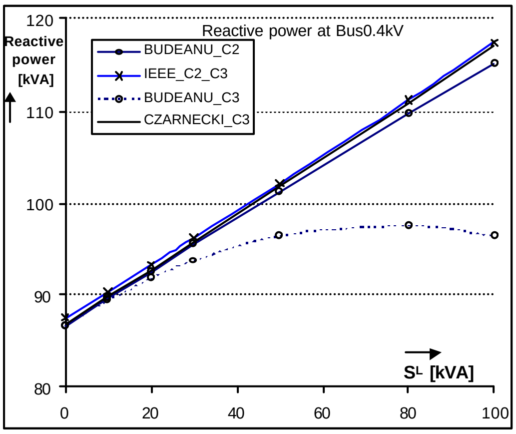

Fig. 6. Reactive power calculated at *Bus0.4kV* for first (Case2) and second (Case3) type of non-linear load.

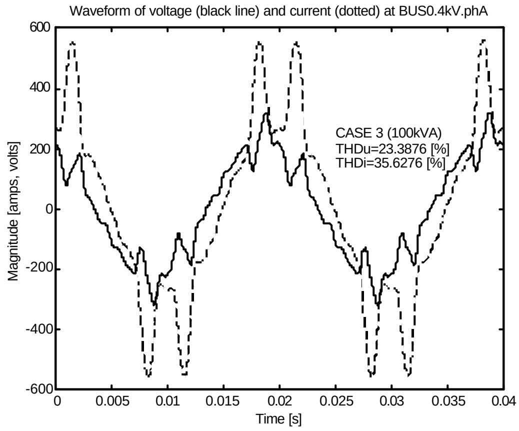

Fig. 7. Waveform of voltage (black line) and current (dotted line) at *Bus0.4kV* in phase *A*, Case 3, S_L=100kVA.

6. CONCLUSIONS

The conclusions can be summarized as follows:

1. From Fig. 3 can be obtained that phase *B* has a smaller *THD* than the other two. This can be easily explained when we notice that there is a greater amount of linear load in phase *B* than in both other phases (see: load position *e* in Tab. 1). The shape of the curve indices the fact that a relatively big amount of many types harmonic producing loads may cause harmonic cancellation due to phase differences and, as a result, a smaller *THD* at the main bus.
2. The second type of non-linear load with two times higher *THDi* (but not of the same harmonic contents) causes much higher voltage harmonic distortion than first one (Fig. 4). It was observed that in both cases bus *HVT* has the lowest values of *THDu*. This confirms the fact, that the transformer impedance has a big influence on the damping of the harmonic distortion until it works in a linear region of its magnetizing characteristics. In fact, during the simulations, the THDu at transformer high voltage bus (HVT) has never exceeded 0.3%.
3. The concept of active power calculation is quite similar for the three theories (Tab. 2), however calculation of reactive power according to Budeanu equation fail (Fig. 6) especially when high harmonic distortion is present in the system (Fig. 7).
4. This study was made to introduce the problem of the presence of high power ratio loads with pulsed power in a distribution system. SuperHarm® come out as a useful software tool for this investigated problem. Further work will concentrate on the measurements in the network, verification of the calculated values and determination of accurate models of the system elements for harmonic propagation in such a distribution system

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