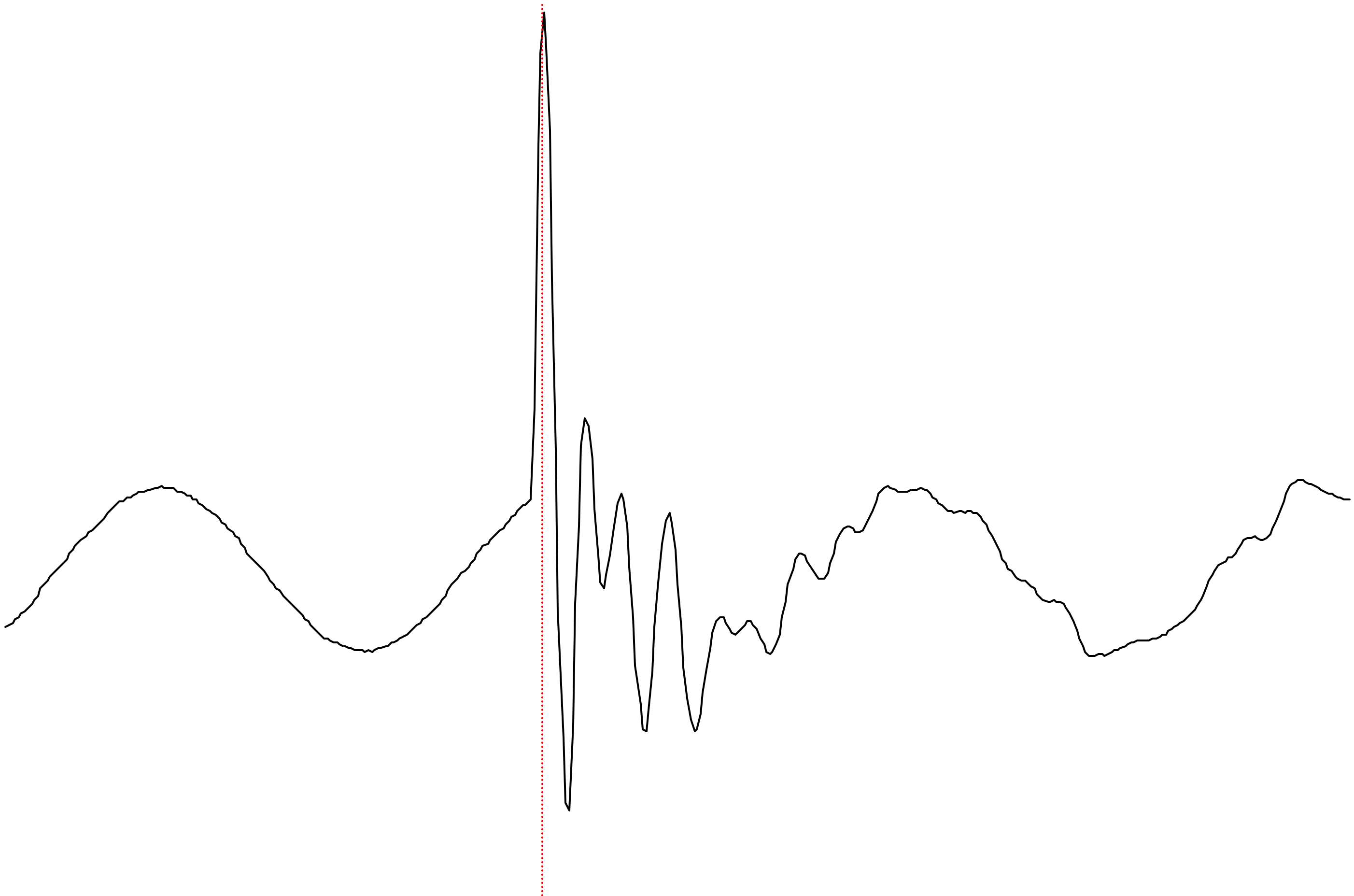
Harmonics and Transients Tech Notes



Issue # 97-3 ***November 1997*** ***Editor: Kathryn Nix***

Project Manager: Huyen Nguyen

In this issue:

Letter from the Project Manager

Dear PATH Members:

The annual PATH Users Group Meeting was a success, and I would like to take a moment to thank all of guest speakers for contributing to the success of this year's meeting. Preparing a presentation takes time, and I know that time is not something that is easily found. So thank you for sharing your knowledge and your time.

I would like to extend a special thank you to Professor Hermann Dommel. Professor Dommel conducted an EMTP workshop, and I know I can speak for the entire group when I say that it was a privilege getting to hear Professor Dommel speak.

Finally, to all who were involved in planning the meeting, thank you for your hard work.

Sincerely,

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THE HARTLEY TRANSFORM FOR THE ANALYSIS OF HARMONIC PROPAGATION IN LINEAR AND NONLINEAR POWER CIRCUITS IN PERIODICAL STEADY STATE

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Abstract
This paper shows a methodology using the Hartley transform for the analysis of harmonic propagation in electrical networks under nonsinusoidal conditions, considering linear and nonlinear elements. At first linear elements are analyzed and a linear Norton equivalent is obtained from nonlinear elements.

Keywords: Hartley Transform, harmonic propagation, nonlinear power circuits, periodical steady state, power quality analysis.

Introduction
Voltage and current waveforms in electrical power systems are frequently nonsinusoidal. The harmonics generation , propagation, effects and solutions have been the principal objectives of Power Quality [1]. Electric Power Quality has six main aspects: Modelling and analysis, instrumentation, sources, solutions, fundamental concepts and effects.

This paper takes place in modelling and linear analysis using the Hartley Transform extended to the nonlinear behavior, taking advantage of its principal characteristic of being of real nature. Other properties of the Hartley transform are [2][8]:

- Exist when the Fourier Transform exist and vice versa.
- It can be obtained from the Fourier Transform.
- The convolution is similar as using the Fourier Transform if one of the two signals is even or odd.
- The Fast Hartley Transform is twice as fast and needs half the memory than the Fourier Transform.

The Hartley Transform
The Hartley Transform of the function f(t) is:

$$H(\boldsymbol{n}) = \int_{-\infty}^{\infty} f(t)\,\mathrm{cas}(\boldsymbol{n}t)\,dt \qquad (1)$$

$$f(t) = \int_{-\infty}^{\infty} H(\nu)\,\mathrm{cas}(\nu t)\,d\nu \qquad (2)$$

where cas(θ)=cos(θ)+sin(θ) and $\nu = 2\pi f$ is the angular frequency in rad/sec.

Equation (1) is the Hartley Transform of f(t) and (2) is the Inverse Hartley Transform of H(ν).

A. ***Discrete Hartley transform***

The Discrete Hartley Transform is expressed as:

$$H(k\Delta\nu) = \frac{1}{N}\sum_{i=0}^{N-1} V(i\Delta T)\,\mathrm{cas}(ik\Delta\nu\Delta T) \qquad k=0,1,\ldots,N-1 \qquad (3)$$

$$V(k\Delta T) = \sum_{i=0}^{N-1} H(i\Delta\nu)\,\mathrm{cas}(ik\Delta\nu\Delta T) \qquad k=0,1,\ldots,N-1 \qquad (4)$$

where (3) is the Discrete Hartley Transform of V(t) and (4) is the Inverse Discrete Hartley Transform.

Where:

N	number of points in the sequence.
T_{total}	total time per cycle (example for 60 cycles, T_{total} = 16.67 ms.)
$\Delta T = \frac{T_{total}}{N}$	time resolution.
$\Delta\nu = \Delta\omega = \frac{2\pi}{N\Delta T}$	frequency resolution.

giving The Fast Hartley Transform (FHT) [2,3].

B. ***The Hartley series***

The Hartley series are defined as:

$$f(t) = \sum_{n=-\infty}^{\infty} S_n\,\mathrm{cas}(n\nu t) \qquad (5)$$

Where:

$$S_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,\mathrm{cas}(n\boldsymbol{n}t)\,dt \qquad (6)$$

the Hartley series coefficients have the following properties:

TABLE I: Coefficient properties.

f(t)	Coefficients
Even: f(t)=f(-t)	$S_n = S_{-n}$
Odd: f(t)=-f(-t)	$S_n = -S_{-n}$

Behavior Of Elements Under Non-sinusoidal Conditions

In general, the load voltage and current under non-sinusoidal conditions are represented by:

$$v(t) = \sum_{m=-\infty}^{\infty} V_m \,\mathrm{cas}(m\boldsymbol{n}t) \qquad (7)$$

$$i(t) = \sum_{n=-\infty}^{\infty} I_n \,\mathrm{cas}(n\boldsymbol{n}t) \qquad (8)$$

where n=m if the load is linear, and n≠m if the load is nonlinear.

A. *Linear elements response*

Passive elements response (resistors, inductors and capacitors) in nonsinusoidal conditions is given by:

Resistor (R) voltage

$$V_R = Ri(t) = \sum_{n=-\infty}^{\infty} RI_n \,\mathrm{cas}(n\boldsymbol{n}t) \qquad (9)$$

Inductor (L) voltage

$$V_R = L\frac{di(t)}{dt} = \sum_{n=-\infty}^{\infty} Ln\boldsymbol{n}I_n \,\mathrm{cas}(-n\boldsymbol{n}t) \qquad (10)$$

Capacitor (C) voltage

$$V_C = \frac{1}{C}\int i(t)dt = \sum_{n=-\infty}^{\infty} \frac{-1}{Cn\boldsymbol{n}} I_n \operatorname{cas}(-n\boldsymbol{n}t) \qquad (11)$$

where the voltage will have the form (7). In matrix form (9), (10), and (11):

$$\begin{bmatrix} \vdots \\ V_{R-2} \\ V_{R-1} \\ V_{R0} \\ V_{R1} \\ V_{R2} \\ \vdots \end{bmatrix} = \begin{bmatrix} R & & & & & & \\ & & & & \tilde{0} & & \\ & & & & & & \\ & & & \ddots & & & \\ & & & & & & \\ & \tilde{0} & & & & & \\ & & & & & & R \end{bmatrix} \begin{bmatrix} \vdots \\ I_{-2} \\ I_{-1} \\ I_0 \\ I_1 \\ I_2 \\ \vdots \end{bmatrix} \qquad (12)$$

$$\begin{bmatrix} \vdots \\ V_{L-2} \\ V_{L-1} \\ V_{L0} \\ V_{L1} \\ V_{L2} \\ \vdots \end{bmatrix} = \begin{bmatrix} & & & & & & \cdot^{\cdot^{\cdot}} \\ & \tilde{0} & & & & 2nL & \\ & & & & nL & & \\ & & & 0 & & & \\ & & -nL & & & & \\ & -2nL & & & & \tilde{0} & \\ \cdot^{\cdot^{\cdot}} & & & & & & \end{bmatrix} \begin{bmatrix} \vdots \\ I_{-2} \\ I_{-1} \\ I_0 \\ I_1 \\ I_2 \\ \vdots \end{bmatrix} \qquad (13)$$

$$\begin{bmatrix} \vdots \\ V_{C-2} \\ V_{C-1} \\ V_{C0} \\ V_{C1} \\ V_{C2} \\ \vdots \end{bmatrix} = \begin{bmatrix} & & & & & & \cdot^{\cdot^{\cdot}} \\ & \tilde{0} & & & & -\frac{1}{2nC} & \\ & & & & -\frac{1}{nC} & & \\ & & & \infty & & & \\ & & \frac{1}{nC} & & & & \\ & \frac{1}{2nC} & & & & \tilde{0} & \\ \cdot^{\cdot^{\cdot}} & & & & & & \end{bmatrix} \begin{bmatrix} \vdots \\ I_{-2} \\ I_{-1} \\ I_0 \\ I_1 \\ I_2 \\ \vdots \end{bmatrix} \qquad (14)$$

in general the Hartley impedance matrix could be represented by (15).

$$Z_H = \begin{bmatrix} R & & & & & & \cdot^{\cdot^{\cdot}} \\ & \ddots & & & & 2X & \\ & & & & X & & \\ & & & & & & \\ & & -X & & & & \\ & -2X & & & & \ddots & \\ \cdot^{\cdot^{\cdot}} & & & & & & R \end{bmatrix} \qquad (15)$$

The matrices (12), (13), (14) y (15) do not have harmonic coupling because current and voltage have the same number of harmonics.

B. ***Linearization of nonlinear elements*** **[4].**

The nonlinear element response is given by the nonlinear equation (16):

$$y = f(x) \qquad (16)$$

where x(t) and y(t) are periodic functions represented by:

$$x(t) = \sum_{h=-\infty}^{\infty} X_h \operatorname{cas}(h\boldsymbol{n}t) \qquad (17)$$

$$y(t) = \sum_{k=-\infty}^{\infty} Y_k \operatorname{cas}(k\boldsymbol{n}t) \qquad (18)$$

If (16) is differentiable, then it is represented by:

$$\Delta y = f'(x_e)\Delta x \qquad (19)$$

Where:

$$\Delta x = \sum_{h=-\infty}^{\infty} \Delta X_h \operatorname{cas}(h\boldsymbol{n}t) \qquad (20)$$

$$\Delta y = \sum_{k=-\infty}^{\infty} \Delta Y_k \operatorname{cas}(k\boldsymbol{n}t) \qquad (21)$$

$$f'(x_e) = \sum_{i=-\infty}^{\infty} C_i \operatorname{cas}(i\boldsymbol{n}t) \qquad (22)$$

substituting (20), (21) and (22) into (19):

$$\sum_{k=-\infty}^{\infty} \Delta Y_k \operatorname{cas}(k\boldsymbol{n}t) = \frac{1}{2}\sum_{i=-\infty}^{\infty}\sum_{h=-\infty}^{\infty} C_i \Delta X_h \left[\operatorname{cas}(i+h)\boldsymbol{n}t + \operatorname{cas}(i-h)\boldsymbol{n}t + \operatorname{cas}(-i+h)\boldsymbol{n}t - \operatorname{cas}(-i-h)\boldsymbol{n}t\right] \qquad (23)$$

Taking h=j and grouping terms for the same harmonic, (23) will be:

$$\underset{k}{\begin{bmatrix} \vdots \\ \Delta Y_{-2} \\ \Delta Y_{-1} \\ \Delta Y_{0} \\ \Delta Y_{1} \\ \Delta Y_{2} \\ \vdots \end{bmatrix}} = \frac{1}{2}\underset{i=k-j}{\begin{bmatrix} \vdots \\ C_{-2-j} \\ C_{-1-j} \\ C_{0-j} \\ C_{1-j} \\ C_{2-j} \\ \vdots \end{bmatrix}} \Delta X_j + \frac{1}{2}\underset{i=k+j}{\begin{bmatrix} \vdots \\ C_{-2+j} \\ C_{-1+j} \\ C_{0+j} \\ C_{1+j} \\ C_{2+j} \\ \vdots \end{bmatrix}} \Delta X_j + \frac{1}{2}\underset{i=-k+j}{\begin{bmatrix} \vdots \\ C_{2+j} \\ C_{1+j} \\ C_{0+j} \\ C_{-1+j} \\ C_{-2+j} \\ \vdots \end{bmatrix}} \Delta X_j - \frac{1}{2}\underset{i=-k-j}{\begin{bmatrix} \vdots \\ C_{2-j} \\ C_{1-j} \\ C_{0-j} \\ C_{-1-j} \\ C_{-2-j} \\ \vdots \end{bmatrix}} \Delta X_j \qquad (24)$$

Grouping (24) :

$$\begin{bmatrix} \vdots \\ \Delta Y_{-2} \\ \Delta Y_{-1} \\ \Delta Y_{0} \\ \Delta Y_{1} \\ \Delta Y_{2} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ C_{(-2,j)} \\ C_{(-1,j)} \\ C_{(0,j)} \\ C_{(1,j)} \\ C_{(2,j)} \\ \vdots \end{bmatrix} \Delta X_j \qquad (25)$$

Where:

$$C_{(k,h)} = \frac{1}{2}\left(C_{k+h} + C_{k-h} + C_{-k+h} - C_{-k-h}\right) \qquad (25.a)$$

for all terms of h, (25) is represented by (26)

$$\begin{bmatrix} \vdots \\ \Delta Y_{-2} \\ \Delta Y_{-1} \\ \Delta Y_{0} \\ \Delta Y_{1} \\ \Delta Y_{2} \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \ddots & \ddots & \vdots & & & \\ \ddots & C_{(-2,-2)} & C_{(-2,-1)} & C_{(-2,0)} & C_{(-2,1)} & C_{(-2,2)} & \\ \ddots & C_{(-1,-2)} & C_{(-1,-1)} & C_{(-1,0)} & C_{(-1,1)} & C_{(-1,2)} & \\ & C_{(0,-2)} & C_{(0,-1)} & C_{(0,0)} & C_{(0,1)} & C_{(0,2)} & \\ & C_{(1,-2)} & C_{(1,-1)} & C_{(1,0)} & C_{(1,1)} & C_{(1,2)} & \ddots \\ & C_{(2,-2)} & C_{(2,-1)} & C_{(2,0)} & C_{(2,1)} & C_{(2,2)} & \ddots \\ & & & \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \Delta X_{-2} \\ \Delta X_{-1} \\ \Delta X_{0} \\ \Delta X_{1} \\ \Delta X_{2} \\ \vdots \end{bmatrix} \qquad (26)$$

in simple form

$$\Delta Y = F \Delta X \qquad (27)$$

Where:

ΔX : is formed by the series coefficients of (20) using the FHT.
ΔY : is formed by the series coefficients of (21) using the FHT.
F: is formed by the series coefficients of (22) using the FHT, and is not a full matrix, depending of the number of harmonics to be analyzed. It is real and symmetric.

if (22) is even then (26) is:

$$\begin{bmatrix} \vdots \\ \Delta Y_{-2} \\ \Delta Y_{-1} \\ \Delta Y_0 \\ \Delta Y_1 \\ \Delta Y_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \ddots & \ddots & \vdots & & & \\ \ddots & C_0 & C_{-1} & C_{-2} & \vdots & & \\ \ddots & C_1 & C_0 & C_{-1} & C_{-2} & \vdots & \\ & C_2 & C_1 & C_0 & C_{-1} & C_{-2} & \\ & \vdots & C_2 & C_1 & C_0 & C_{-1} & \ddots \\ & & \vdots & C_2 & C_1 & C_0 & \ddots \\ & & & \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \Delta X_{-2} \\ \Delta X_{-1} \\ \Delta X_0 \\ \Delta X_1 \\ \Delta X_2 \\ \vdots \end{bmatrix} \qquad (27.a)$$

if (19) is linearized around (x_b , y_b) where $\Delta X=X-X_b$ and $\Delta Y=Y-Y_b$, then (27) is given by:

$$Y=FX+Y_N \qquad (28)$$

Where:

$$Y_N=Y_b-FX_b \qquad (29)$$

C. Use of (28) and (29) to represent the saturation current of an inductive element

The saturation current of an inductive element is represented by the nonlinear equation:

$$i=f(\varphi) \qquad (30)$$

Applying (28) and (29) to (30):

$$I = F\Psi + I_N \qquad (31)$$

$$I_N = I_b - F\Psi_b \qquad (32)$$

with:

$$V = \dot{\Psi} = D\Psi \qquad (33)$$

Where:

$$D=\begin{bmatrix} & & & & & \ddots \\ & \underset{\sim}{0} & & & 2n & \\ & & & n & & \\ & & 0 & & & \\ & -n & & & & \\ -2n & & & & \underset{\sim}{0} & \\ \ddots & & & & & \end{bmatrix} \qquad (34)$$

substituting (33) into (31) and (32)

$$I = BV + I_N \qquad (35)$$

$$I_N = I_b - BV_b \qquad (36)$$

where $B=FD^{-1}$ has the same characteristics of F.

Equations (35) and (36) represent the Norton equivalent of Figure 1.

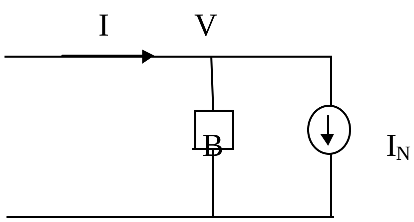


Fig. 1 Norton equivalent

Linear Solution To The Harmonic Propagation Problem

The current injection method works for linear network models. Near the harmonics sources the injection current waveform is known; and the harmonic propagation problem is solved by superposition, solving for each harmonic as is shown by the next system of linear equations [5]:

$$I_h=Y_hV_h \qquad (37)$$

Other method is to obtain the equivalent impedance for each harmonic between the node with a harmonic source and the node where the effect of the harmonics is to be determined. Figure 2 shows this method [6]:

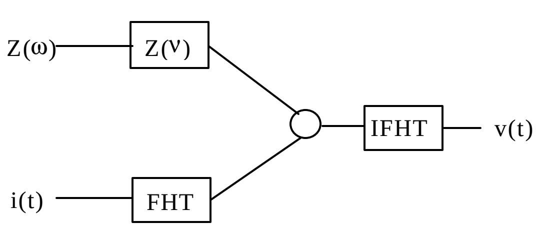


Fig. 2 Flow diagram, linear solution of harmonic propagation.

Iterative Solution To The Harmonic Propagation Problem

Representing the electric network by two equivalents as in Figure 3.

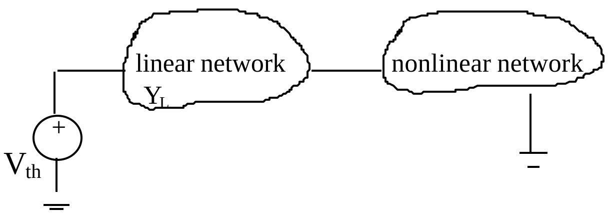


Fig. 3 Electric network equivalent.

the linear network portion is represented by Hartley admittances and the nonlinear network by Norton equivalents.

The solution method is based in Figure 4.

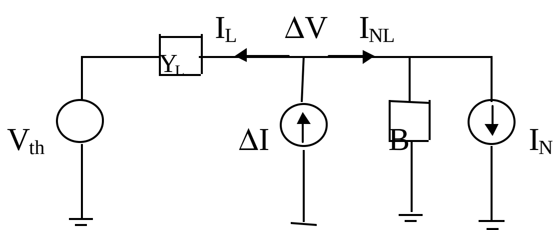


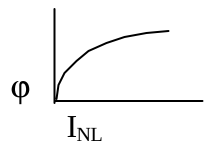
Fig. 4 Equivalent network[7] for the iterative process.

If in each iteration (35) and (36) are considered (which represent the Norton equivalent) $I_b=I$ and $V_b=V$, this means linearizing in each iteration, then the current I can be obtained from (30) using (33). The matrix B can be used only, to speed up convergence to the solution.

Proposed method:

1. Obtain (from a load flow study at fundamental frequency) the voltage V(t), where the linear and nonlinear network are joined.
2. Obtain $V(\nu)$ using the FHT.
3. Compute $I_L(\nu)=Y_L(\nu)\{V(\nu)-V_{th}(\nu)\}$
4. Calculate $I_{NL}(t)$
   a. Compute $\varphi(\nu)=D^{-1} V(\nu)$
   b. Obtain $\varphi(t)$ using the IFHT

c. Obtain $I_{NL}(t)=f(\varphi(t))$



d. Obtain $f'(t)=\dfrac{I_{NL}(t+\Delta t)-I_{NL}(t)}{\varphi(t+\Delta t)-\varphi(t)}$

e. Obtain $F'(\nu)$ using the FHT, and build F.

f. Obtain $B=FD^{-1}$

5. Obtain $I_{NL}(\nu)$ using the FHT.
6. Compute $\Delta I(\nu)=I_L(\nu)+I_{NL}(\nu)$
7. Compute $\Delta V(\nu)=\{Y_L(\nu)+B\}^{-1}\Delta I(\nu)$
8. Obtain the new value of $V(\nu)=V(\nu)-\Delta V(\nu)$
9. Obtain V(t) using the IFHT.
10. Back to the point 3 if $|\Delta V(\nu)|>\varepsilon$

In the above method it is sufficient to compute only once matrix B because in each iteration it has minor changes.

Other simplifications are obtained when the nonlinear network introduces a low distorsion grade, then point 7 can be substituted by $\Delta V(\nu)=Y_L(\nu)^{-1}\Delta I(\nu)$, and the same solution is obtained with no more than two extra-iterations, without making necessary the construction of matrix B.

Results

A. *Linear method (current injection method)*

MATLAB® was used to implement the flow diagram of Figure 2, a distribution network of 8 nodes [6], Appendix A, is used to compute the voltage waveform that appears at node 1 caused by a harmonic current injection in node 8.

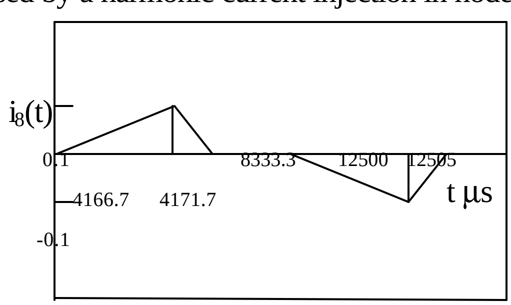


Fig. 5 Current injection waveform in the node 8.

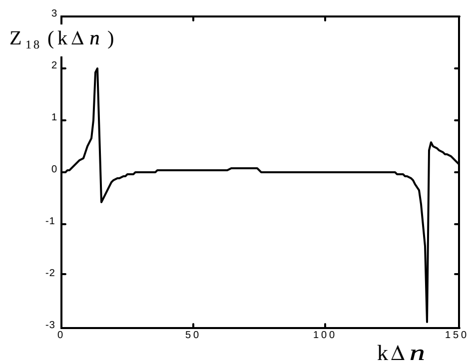
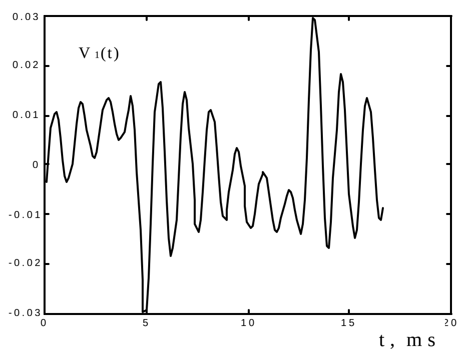
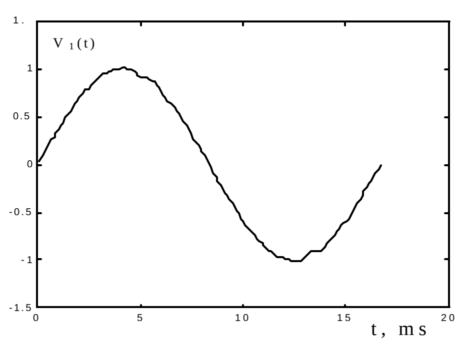


Fig. 6 Hartley transform, impedance Z_{18}



a)



b)

Fig. 7 Voltage in the node 1, a) Harmonic components caused by the current injection at node 8. b) Harmonic components plus fundamental frequency of 60 Hz.

B. ***Iterative method***

The proposed method was used to compute the current response of a nonlinear circuit:

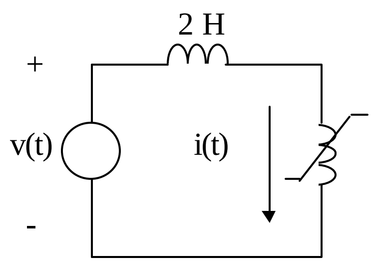
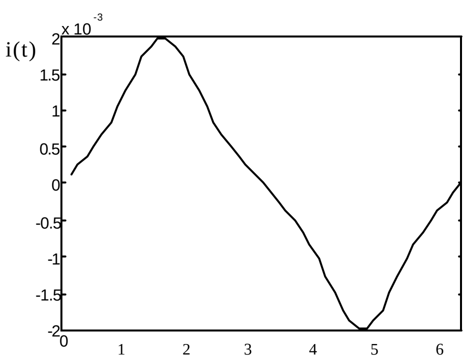


Fig. 8 Nonlinear circuit

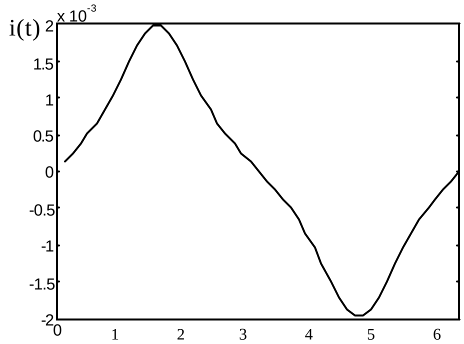
where the nonlinear element response is given by:

$$i(t) = f(\boldsymbol{j}) = \frac{1}{1000}\left(\boldsymbol{j} + \boldsymbol{j}^{5}\right)$$

Figures 9 and 10 show the current i(t) response under different voltages v(t), SIMNON was used to compare results.

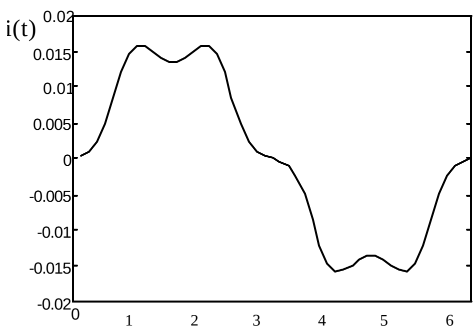


a) with matrix B, I2 iterations.

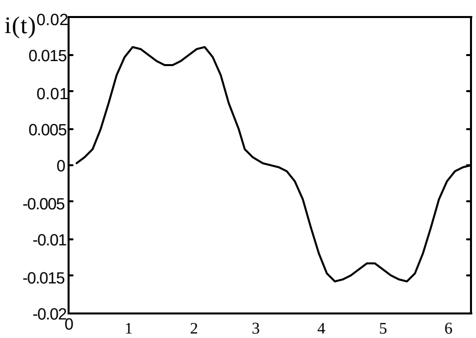


b) without matrix B, 3 iterations.

Fig. 9 v(t)=cos(ωt), using the proposed method.



a) with matrix B, 5 iterations.



b) without matrix B, 5 iterations.

Fig. 10 v(t)=2cos(ωt)+cos(3ωt), using the proposed method.

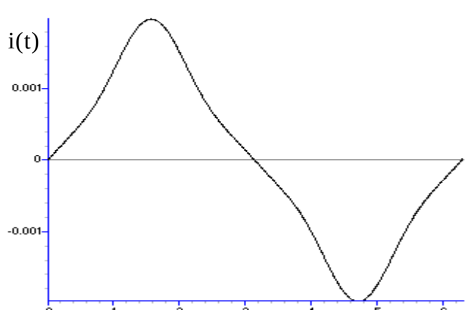


Fig. 11 v(t)=cos(ωt) using SIMNON

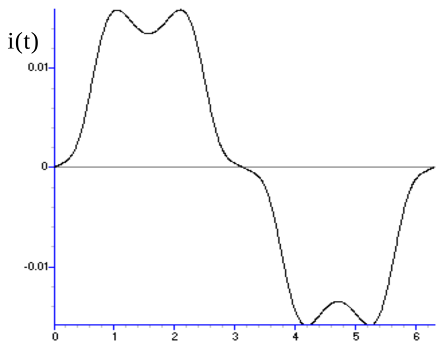


Fig. 12 v(t)=2cos(ωt)+cos(3ωt), using SIMNON

Conclusions

- It was showed that the Hartley Transform can be used to to represent nonlinear elements as a linearized Norton equivalent.
- The electric network is represented by real numbers only.
- The matrices dimensions are the same than using the Fourier transform, with the advantage of working with real instead of complex matrices.
- The B matrix is the only one that has shows harmonic coupling and has a (2h)x(2h) dimension, where h is the number of harmonics to be considered. If B is builded in the Hartley domain it is a real and symmetric matrix.

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Appendix A

The test system used is shown in Figure 13, N = 150 and T_{total} =16.6 ms were used.

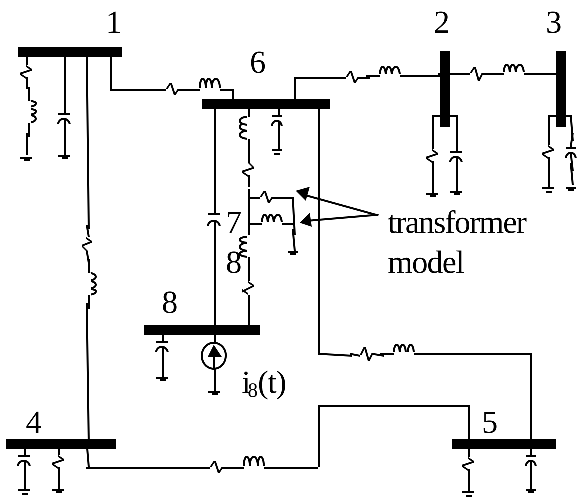

Fig. 13 Distribution circuit

TABLE II: Impedance data in p.u of the distribution circuit, (h=1 for 60Hz).

branch	branch	branch
value	value	value
1-6	2-0	5-0
0.001+jh0.01	1	1
1-4	3-0	6-8 -
0.001+jh0.01	2	j200/h
1-0 -	3-0 -	6-7
j50/h	j100/h	0.01+jh0.1
1-0	4-5	6-0 -
0.1+jh0.1	0.001+jh0.001	j40/h
2-3	4-0 -	7-8
0.001+jh0.01	j50/h	0.01+jh0.1

2-6 0.001+jh0.01 2-0 - j50/h	4-0 1 5-6 0.001+jh0.01 5-0 - j100/h	7-0 100 7-0 jh10 8-0 - j40/h

Biographies

Manuel Madrigal Martínez. Was born in Purépero Mich; México in September 7, 1969. He received his B.Sc. in EE with honors at the Instituto Tecnológico de Morelia in 1993. He obtained the “Adolfo Lopéz Mateos” award by the DGIT (General Direction of Tech. Institutes). He received his M.Sc at Doctoral Program in Electrical Engineering at Universidad Autónoma de Nuevo León in 1996. At present hi is Professor of the Electric and Electronic Department at the Instituto Tecnológico de Morelia.

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He was Director of the Electrical Engineering School of the UMSNH and has tought in the Inst. Tec. de Morelia. His principal interests are in modelling and analysis of power systems, the efficient operation and control of electric power systems.

Understanding Wavelet Transforms

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Abstract

The purpose of this paper is to introduce the so-called wavelet transforms and their basic properties to PATH members. In preparing this paper, no prior knowledge of wavelet theory is assumed, but the Fourier transform only. The presentation of the paper starts off with the familiar Fourier transform, and followed by the mathematical signal representation, before detailing the wavelet transform. The message in this paper is that the wavelet transform is a complementary tool to analyze transient or non-stationary signals, and it is not intended to replace the well-established Fourier transform in analyzing stationary (steady state) signals. Various applications of wavelets in power systems are summarized in this paper.

Introduction

Wavelet analysis has been a popular signal analysis technique in recent years. It has opened up new avenues of research and applications in various areas. For example, wavelet transforms have been extensively used in signal processing community for data and image compressions, feature extractions, non-stationary signal analysis, and speech and image processing to mention a few. In mathematics, wavelet transforms have been utilized to solve complex linear algebra problems and pseudo-differential equations, and have been closely tied to the approximation theory. Wavelet analysis has also been very useful in fluid dynamics and turbulence studies. Various applications in other areas such as biomedical signal analysis to vibration analysis have benefited from the wavelet analysis as well.

In power systems analysis, wavelet transforms have started to gain popularity. There are approximately half a dozen papers published in IEEE Transactions on Power Delivery to date. These papers largely deal with transient analysis and power quality event detection. As in other areas of applications, the applications of wavelet analysis in power systems have been enthusiastic since it solves some problems that previously cannot be solved using Fourier-based techniques. However, it should be noted that the wavelet transform is not intended to replace the Fourier transform. It is a complementary tool to the Fourier transform in signal analysis. Both transforms have their own unique strengths in solving engineering problems. Their strengths are tied with their mathematical properties; thus, some problems are better solved with one technique but not with the other.

In this article, the wavelet transform will be explained from a novice point of view; however, some basic familiarity with the Fourier transform is assumed. Properties of the wavelet transform will be discussed as well as the difference between the wavelet and Fourier transforms. Current and potential applications of wavelet analysis on power quality are also reviewed.

Mathematical Signal Representations

In many engineering problems, measurements are often taken with respect to time, spatial location, a combination of both, or some other physical measures. This suggests that measurements are taken in specific domains. For example, in power quality analysis, voltage and current variations are taken with respect to time resulting in voltage and current waveforms in the time domain, respectively. In other area of studies such as in aerodynamics, longitudinal velocity of a turbulent supersonic flow is measured spatially, resulting in a spatial velocity profile at a particular time instant in the spatial domain. For both cases the signals are voltage and current waveforms and spatial velocity profile, respectively.

Measured signals carry information associated with the physical nature of the system. However, the information or signal characteristics may not be evident because they may not be located in the measured domain, i.e., the time or spatial domains. Therefore, in an attempt to reveal signal characteristics, a given measured signal is often represented in a domain in which the information resides.

From this point on, we will use a voltage waveform as an example of our signal. Since voltage is measured with respect to time, it is also called a voltage time-series. Figure 1 below shows a voltage notching waveform in the time domain that was recorded using a power monitor.

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Figure 1. Voltage notching waveform in the time domain recorded using a power monitor. The measured signal is indeed a voltage time-series, i.e., voltage vs. time

Given a time-domain signal shown above and without a prior knowledge in power quality and harmonics, it would be difficult to describe what the signal is all about. Oftentimes, a Fourier transformation is performed to reveal its frequency components. By doing this, we should say that a time domain signal (i.e., the voltage time-series) is represented in the frequency domain. The objective of signal representations is to reveal information or specific signal characteristics that may reside in particular domains. In this case, the information of interest resides in the frequency domain, and the Fourier transform is the bridge connecting the two domains. This example is a special case of the well known time domain signal representation using the Fourier transform.

In the following, we will present a more general case of signal representations. Let $x(t)$ be a given signal measured in the t domain and let there exist a transformation kernel $g_u(t)$. We now wish to represent the t domain signal $x(t)$ in the u domain. The bridge connecting the two domains is the aforementioned function $g_u(t)$. An inner product is used as a vehicle to represent signal $x(t)$ in the u domain. An inner product is defined as follows:

$$F(u) = \langle x(t), g_u(t) \rangle = \int_{-\infty}^{\infty} x(t) g_u^*(t) dt \,, \qquad (1)$$

where $g_u^*(t)$ is a complex conjugate of $g_u(t)$. $F(u)$ is indeed signal $x(t)$ in the u domain.

In the Fourier transform case, $x(t)$ is in the time domain and the transformation kernel is $g_u(t) = e^{j2pft}$. Thus, the frequency domain signal of $x(t)$, $F(f)$, is as follows:

$$F(f) = \langle x(t), e^{j2pft} \rangle = \int_{-\infty}^{\infty} x(t) e^{-j2pft} dt \,. \qquad (2)$$

Reader should easily recognize that Eq. (2) is the familiar Fourier transform. The above Fourier's transformation kernel is a sinusoidal function that is characterized by frequency or number of cycles per second.

The Fourier transform is especially suitable in analyzing stationary signals (signals whose properties do not change in time) because the building block of such signals are characterized by frequency which is also the characteristic of the Fourier's transformation kernel. Thus for such signals, the Fourier transform is the most appropriate tool to use. In the next section, we will show another variant of the Fourier transform and the wavelet transform and discuss their properties.

3. The Short-time Fourier Transform

It is not uncommon to come across signals that are non-stationary, i.e., properties of the signal change with time. Well-known examples for such signals are transients, intermittent, and impulsive signals. These signals in power systems are typically originated in power quality disturbances. The Fourier transform is generally not suited for analyzing non-stationary signals because the desired information is located in both time and frequency domains. For such signals, local characteristic changes are not well represented, and the corresponding information is spread out over the entire frequency domain. This is obvious because the Fourier transform as shown in Eq.(2) does not provide temporal information.

Another example of a signal representation that includes time and frequency representations is the short-time Fourier transform. This transform is intended to alleviate the non-local problem; thus, the time-dependent variable is introduced into Eq.(2). The non-stationary signal *x(t)* is multiplied with an appropriate window function *w(t)* centered at temporal location τ. The resulting signal is then assumed stationary within the window and Fourier transformed. Such a transformation is known as a short-time Fourier transform shown below:

$$X_{STFT}(f,\boldsymbol{t}) = \int x(t)w^*(t-\boldsymbol{t})e^{-j2pft}dt, \qquad (3)$$

where the basis function is $g_{t,f}(t) = w(t-\boldsymbol{t})e^{j2pft}$. In other words, $X_{STFT}(f,\boldsymbol{t})$ is the Fourier transform of *x(t)* windowed with $w(t)$ shifted by τ. This modified version of the FT has the capability to provide a time-frequency description of the signal. In addition, note that the time-support of the window function *w(t)* is constant for all frequencies, and the basis function $g_{t,f}(t) = w(t-\boldsymbol{t})e^{j2pft}$ is in fact a constant-width modulated function with frequency *f*. Thus, once the window function $w(t)$ has been chosen, the analysis resolution in time and frequency domains is constant. As a result, the ability to capture the dynamic of the signal characteristics is greatly dependent upon the choice of the window function $w(t)$.

4. **The Wavelet Transform**

In both Fourier and short-time Fourier transforms, the transformation kernels are characterized by frequency. A different method to analyze non-stationary signals that does not use frequency is the wavelet transform. The wavelet transform seeks to represent a time domain signal in the time-scale domain. For this reason the wavelet transform analysis is technically called time-scale signal analysis.

In the time-scale signal representation, a given time domain signal $x(t)$ is represented by an inner product with a wavelet, thereby the wavelet transform. It is defined as follows:

$$W_x(a,\tau) = \left\langle x(t), \psi_{a,\tau}(t) \right\rangle = \int_{-\infty}^{\infty} x(t)\psi^*_{a,\tau}(t)dt, \qquad (4)$$

where $\psi_{a,\tau}(t)$ is

$$\psi_{a,\tau}(t) = |a|^{-1/2}\psi\left(\frac{t-\tau}{a}\right), \qquad (5)$$

$\psi(t)$ is a mother wavelet whereas $a \in R, a \neq 0$ is the dilation or scale number, and $\boldsymbol{t} \in R$ is the time-shift parameter. *R* is a real continuous number system. The asterisk denotes a complex conjugate operation.

By comparing Eqs. (2), (3), and (4), it should be clear now that the fundamental difference between Fourier-based transforms and the wavelet transform is in their transformation kernels. This fundamental difference leads to a substantial implication in the use of the two transforms (i.e., Fourier and wavelet). As we have pointed out earlier the Fourier transform is better able to analyze stationary signals, and we shall see later, the wavelet transform is better able to analyze non-stationary signals.

Unlike a sinusoidal function which oscillates forever, a wavelet function is a “small” wave that oscillates for a “short” period of time. Mathematically, a function is called a wavelet if it satisfies the following condition:

$$\int_{-\infty}^{\infty} y(t)dt = 0 . \qquad (6)$$

In other words, a wavelet is a zero-mean function and must decay to zero at $\pm\infty$.

The simplest function that can be called a wavelet is a square function as shown in Fig. 2(a). This wavelet is called Haar wavelet. It is defined as follows:

$$\begin{aligned} y(t) &= 1 \qquad 0 \le t < 0.5 \\ &= -1 \qquad 0.5 \le t < 1 \end{aligned} \qquad (7)$$

The above square function oscillates for one cycle only, and it satisfies Eq. (6) because the area under the square function is equal to zero, or it is zero-mean. A sinusoidal function is certainly not a wavelet because it is not a small wave (its value does not decay to zero at $\pm\infty$), although the area under the sinusoidal function is zero.

The attributes of a wavelet transform are dilation and translation. When a wavelet is dilated, its width gets wider but its magnitudes get smaller. Using Eq.(5), let us assume that the dilation factor of the Haar wavelet in Eq.(7) is $a = 1$ or scale 1, and let it be the mother wavelet. Thus at scale 2 ($a = 2$), the Haar wavelet now becomes $y_{2,0}(t) = \frac{1}{\sqrt{2}} y(\frac{t}{2})$. Similarly, the Haar wavelet at scale 3 is $y_{3,0}(t) = \frac{1}{\sqrt{3}} y(\frac{t}{3})$. Figure 2b and 2c show the Haar wavelet at scales 2 and 3, respectively. As one can see, the time support is wider at higher scales, which means the corresponding wavelet loses its time resolution, however, its frequency resolution is now better because of the Heisenberg uncertainty principle. Thus at lower scales, the wavelet transform has a better time resolution, whereas at higher scales, it has a better frequency resolution. This is the reason why wavelets are more suitable to analyze transients or non-stationary signals.

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Figure 2: The simplest possible wavelet, a square function. (a) A square function at scale 1, (b) and (c) are scales 2 and 3 of the wavelet function. Their amplitudes are $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{3}}$, respectively.

The Haar wavelet is not widely utilized because of its discontinuity. However, there are many other wavelet functions that are more useful and widely utilized: Morlet wavelet, Mexican hat wavelet, Daubechies wavelets, etc.

Another attribute of the wavelet transform is translation or time-shift. In this case a wavelet function is translated across time t for every scale number a. Figure 3 illustrates a pictorial dilation and translation procedure in the wavelet transform. The wavelet function is a Mexican hat function for its resemblance to a Mexican hat. A hypothetical signal $x(t)$ is first windowed by a wavelet at scale $a = a_1$ at $\boldsymbol{t} = 0$. This $a = a_1$ wavelet then sweeps the entire signal $x(t)$. In the upper right of Fig. 3, it is shown that $a = a_1$ wavelet at $\boldsymbol{t} = \boldsymbol{t}_0$ location. The same procedure is then repeated for $a = a_2$ wavelet. The lower right of the figure shows that $a = a_2$ wavelet at $\boldsymbol{t} = \boldsymbol{t}_1$ location. Note that $a = a_2$ wavelet is a dilated version of $a = a_1$ wavelet.

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Figure 3. A pictorial operation of the wavelet transform; its transformation kernel is based on translation and dilation.

5. **Continuos and discrete wavelet transforms**

Based on the dilation procedure, the wavelet transform is divided into the continuous and discrete types. In the continuous wavelet transform, the mother wavelet is dilated continuously over the time axis such that the family of analyzing wavelet forms an over-complete basis in $L^2(R)$ (it is a Hilbert space where every signal has a finite energy, defined as $\langle x(t), x(t) \rangle = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$). On the other hand, the mother wavelet in the discrete wavelet transform is dilated discretely. In other words, the dilation or scaling parameter a for the continuous wavelet transform varies continuously; whereas, in the discrete case, the scaling parameter a varies discretely. Figure 4 illustrates the pictorial representation of the continuous and discrete wavelet transforms. One can think the dilation or scaling parameter as a knob in a radio. If the knob turns continuously to any position, then it is a continuous wavelet transform. If it turns only to certain predefined positions, then it is a discrete wavelet transform.

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Figure 4. The difference between (a) continuous and (b) discrete wavelet transforms. The "knob" (i.e., dilation parameter *a*) in the continuous one turns to any position, whereas the discrete one turns to a certain predefined position only.

The wavelet in Eq.(5) is a valid expression for continuous wavelets because $a \in R$. For a discrete wavelet, a given wavelet is discretized in a and $\boldsymbol{t}$, by selecting $a = a_0^m$ and $\boldsymbol{t} = n\boldsymbol{t}_0 a_0^m$, where a_0 and $\boldsymbol{t}_0$ are fixed constants with $a_0 > 1,\ \tau_0 > 0,\ m, n \in Z,$ and Z is a set of integer. Then, the discretized wavelet becomes

$$\psi_{m,n}(t) = \left|a_0\right|^{-m/2} \psi\left(\frac{t - \ n\tau_0 a_0^m}{a_0^m}\right). \qquad (7)$$

The discrete wavelet transform is then given by

$$DWT_x(a, \boldsymbol{t}) = \int_{-\infty}^{\infty} x(t) \boldsymbol{y}_{m,n}^{*}(t) dt \,.$$

Within the discrete wavelet transform family, there are several other types such as the well-known orthonormal-dyadic wavelet transforms, bi-orthogonal wavelet transforms, spline wavelets, and so on. Although they are different kind of wavelet transforms, their fundamental properties are identical. However, their implementations may be quite different.

Avid readers can extend their reading on wavelet transforms for a more complete coverage. Three excellent overview of the wavelet transform from the signal processing and mathematical point of view [1,2,3] are given below.

6. **Applications in Power Systems and Conclusion.**

From the above presentation of wavelet transforms, it should be clear that the principal application of wavelet transforms will be in transient analysis. Wavelet transforms possess beautiful properties that are suitable for such applications. Transformation kernels of the wavelet transform are multi-resolution because it is based on dilation. Thus, the wavelet transform possesses a high time resolution when the wavelet is narrow in the time. However, the wavelet transform also possesses a high frequency resolution when the wavelet is dilated in the time domain.

Along this line, several papers have been published in this subject. The potential applications in power system transients were proposed in [4,6]. The application of wavelets in power distribution relaying is studied in [7]. The use of the wavelet transform in detecting power quality events and compressing power quality data are detailed in [5,8] respectively. Automatic power quality event identification using the wavelet transform and artificial neural network are proposed and implemented in [9].

Future potential application in power systems may include transient analysis, time-varying harmonics, and fault location detector, and so on. The wavelet transform is a complementary signal analysis tool to analyze non-stationary signals, and, thus, it is not intended to replace well-established techniques such as the Fourier transform in analyzing stationary signals. The wavelet transform has presented itself as a new technique that is useful in power systems, especially in the transient and power quality analysis. It will continue and become an important tool in analyzing suitable problems in power systems.

Further Reading:

Tutorial material on wavelets:

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