SuperHarm Benchmarking - BRANCH Modeling Equations

Electrotek Concepts - 6/22/98, EWG/TEG

This document illustrates the equations that define the operation of the BRANCH model.

BRANCH Model Verification:

$r := 0.004 \quad x := 0.02$

$Length := 500$

Case 1a: X/R constant = yes and length = 500ft.

$R := r \cdot Length \quad R = 2$

$X := x \cdot Length \quad X = 10 \qquad Leq := \frac{X}{2 \cdot \pi \cdot 60} \qquad Leq = 0.027$

$Z := R + X \cdot j \quad Z = 2 + 10i$

$h := 1 .. 20$

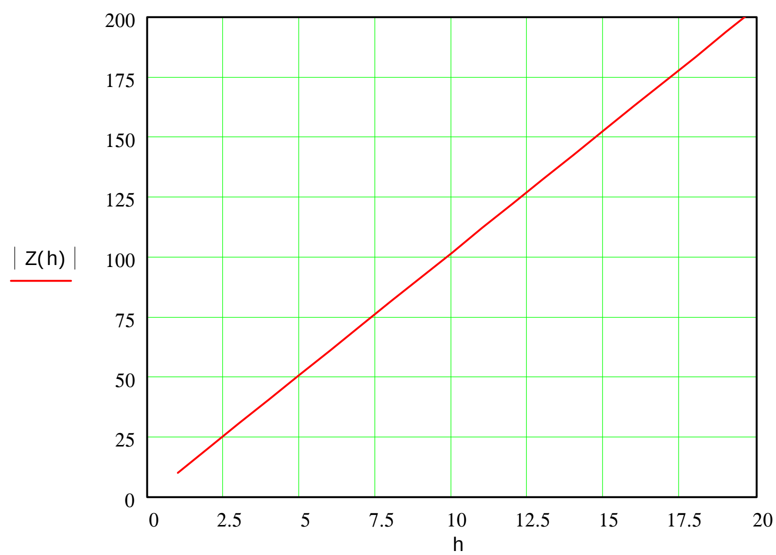
at 60 Hz:

$|Z| = 10.198 \qquad arg(Z) = 78.69 \circ deg$

$R(h) := h \cdot R$

$X(h) := 2 \cdot \pi \cdot 60 \cdot h \cdot Leq \qquad Z(h) := R(h) + X(h) \cdot j$

h	$\lvert Z(h) \rvert$	$(arg(Z(h))) \cdot \frac{180}{\pi}$
1	10.198	78.69
2	20.396	78.69
3	30.594	78.69
4	40.792	78.69
5	50.99	78.69
6	61.188	78.69
7	71.386	78.69
8	81.584	78.69
9	91.782	78.69
10	101.98	78.69
11	112.178	78.69
12	122.376	78.69
13	132.575	78.69
14	142.773	78.69
15	152.971	78.69
16	163.169	78.69
17	173.367	78.69
18	183.565	78.69
19	193.763	78.69
20	203.961	78.69



SuperHarm Benchmarking - BRANCH Modeling Equations

Electrotek Concepts - 6/22/98, EWG/TEG

This document illustrates the equations that define the operation of the BRANCH model.

BRANCH Model Verification:

$r := 0.004 \quad x := 0.02$

$Length := 500$

Cases 1b&1c: X/R constant = no and length = 500ft.

$R := r \cdot Length \quad R = 2$

$X := x \cdot Length \quad X = 10 \qquad Leq := \frac{X}{2 \cdot \pi \cdot 60} \qquad Leq = 0.027$

$Z := R + X \cdot j \quad Z = 2 + 10i$

at 60 Hz:

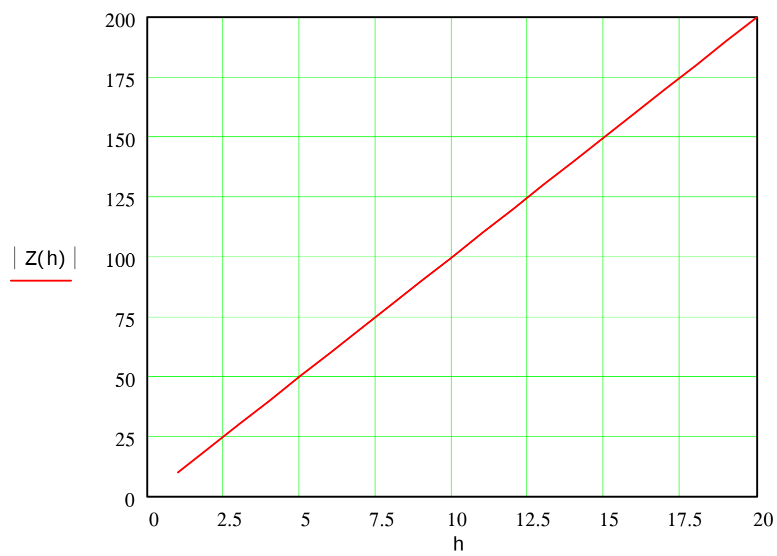
$|Z| = 10.198 \qquad arg(Z) = 78.69 \cdot deg$

$h := 1 .. 20$

$X(h) := 2 \cdot \pi \cdot 60 \cdot h \cdot Leq$

$Z(h) := R + X(h) \cdot j$

h	$\|Z(h)\|$	$(arg(Z(h))) \cdot \frac{180}{\pi}$
1	10.198	78.69
2	20.1	84.289
3	30.067	86.186
4	40.05	87.138
5	50.04	87.709
6	60.033	88.091
7	70.029	88.363
8	80.025	88.568
9	90.022	88.727
10	100.02	88.854
11	110.018	88.958
12	120.017	89.045
13	130.015	89.119
14	140.014	89.182
15	150.013	89.236
16	160.012	89.284
17	170.012	89.326
18	180.011	89.363
19	190.011	89.397
20	200.01	89.427



SuperHarm Benchmarking - BRANCH3
Modeling Equations

Electrotek Concepts - 6/9/98, EWG/TEG

This document illustrates the equations that define the operation of the BRANCH3 model.

BRANCH3 Model Verification:

Cases 2a, 2b, 2c, & 2d:

Input Data (Units per Feet):

	Resistance (Ohms)	Reactance (Ohms @ 60 Hz)
Zero Sequence	$R_0 := 0.080$	$X_0 := 0.600$
Positive Sequence	$R_1 := 0.024$	$X_1 := 0.110$
Frequency (Hertz)	$f := 60$	
Length (feet)	$d := 200$	

$\omega := 2\cdot\pi\cdot f$ $\qquad L := \frac{X}{\omega}$ $\qquad L = \begin{bmatrix} 1.592\cdot 10^{-3} \\ 2.918\cdot 10^{-4} \end{bmatrix}$

$z(\omega) := R + j\cdot\omega\cdot L$ $\qquad z(\omega) = \begin{bmatrix} 0.08 + 0.6i \\ 0.024 + 0.11i \end{bmatrix}$

$Z(d,\omega) := z(\omega)\cdot d$ $\qquad Z(d,\omega) = \begin{bmatrix} 16 + 120i \\ 4.8 + 22i \end{bmatrix}$

$Zps(d,\omega) := \frac{Z(d,\omega)_0 + 2\cdot Z(d,\omega)_1}{3}$ $\qquad Zps(d,\omega) = 8.533 + 54.667i$

$Zpm(d,\omega) := \frac{Z(d,\omega)_0 - Z(d,\omega)_1}{3}$ $\qquad Zpm(d,\omega) = 3.733 + 32.667i$

Build the primative admittance matrix

$$YZp(d,\omega) := \begin{bmatrix} Zps(d,\omega) & Zpm(d,\omega) & Zpm(d,\omega) \\ Zpm(d,\omega) & Zps(d,\omega) & Zpm(d,\omega) \\ Zpm(d,\omega) & Zpm(d,\omega) & Zps(d,\omega) \end{bmatrix}^{-1}$$

Calculate the driving point impedance:

$$Zdp(d,\omega) := YZp(d,\omega)^{(-1)}$$

$$Zdp(d,\omega) = \begin{bmatrix} 8.533+54.667i & 3.733+32.667i & 3.733+32.667i \\ 3.733+32.667i & 8.533+54.667i & 3.733+32.667i \\ 3.733+32.667i & 3.733+32.667i & 8.533+54.667i \end{bmatrix}$$

Build an one amp positive sequence current injection source:

$$I := \begin{bmatrix} \left(\cos\left(0\cdot\frac{\pi}{180}\right) + j\cdot\sin\left(0\cdot\frac{\pi}{180}\right)\right)\cdot 1 \\ \left(\cos\left(-120\cdot\frac{\pi}{180}\right) + j\cdot\sin\left(-120\cdot\frac{\pi}{180}\right)\right)\cdot 1 \\ \left(\cos\left(120\cdot\frac{\pi}{180}\right) + j\cdot\sin\left(120\cdot\frac{\pi}{180}\right)\right)\cdot 1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 \\ -0.5-0.866i \\ -0.5+0.866i \end{bmatrix}$$

Solve the equation:

$$Vo(d,\omega) := Zdp(d,\omega)\cdot I \qquad Vo(d,\omega) = \begin{bmatrix} 4.8+22i \\ 16.653-15.157i \\ -21.453-6.843i \end{bmatrix}$$

Convert to polar coordinates:

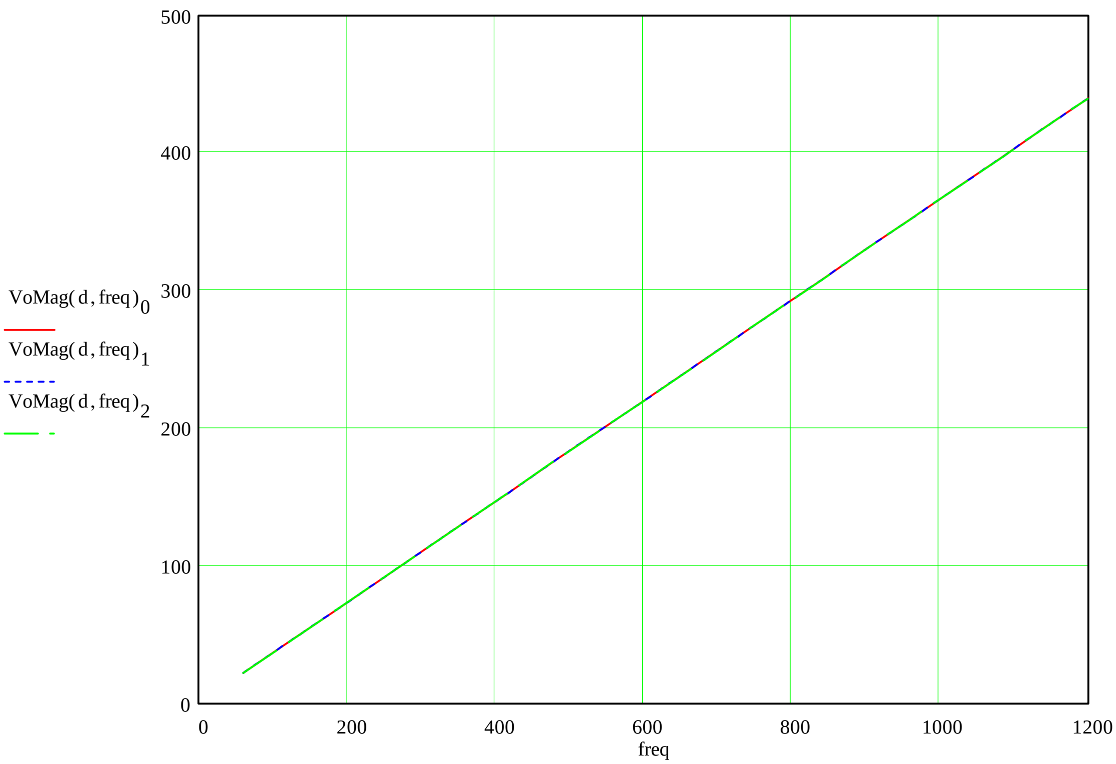
$$VoMag(d,f) := \overrightarrow{\left|Vo(d,2\cdot\pi\cdot f)\right|} \qquad VoAng(d,\omega) := \overrightarrow{\frac{180}{\pi}\cdot angle(Re(Vo(d,\omega)),Im(Vo(d,\omega)))}$$

$$VoMag(d,f) = \begin{bmatrix} 22.518 \\ 22.518 \\ 22.518 \end{bmatrix} \qquad VoAng(d,\omega) = \begin{bmatrix} 77.692 \\ 317.692 \\ 197.692 \end{bmatrix}$$

Perform a frequency sweep of the solution for recieving end voltage:

$freq := 60, 120 .. 1200$

$freq$	$VoMag(d, freq)_1$	$VoAng(d, freq \cdot 2 \cdot \pi)_0$	$VoMag(d, freq)_0$
60	22.518	77.692	22.518
120	44.261	83.774	44.261
180	66.174	85.84	66.174
240	88.131	86.878	88.131
300	110.105	87.501	110.105
360	132.087	87.917	132.087
420	154.075	88.215	154.075
480	176.065	88.438	176.065
540	198.058	88.611	198.058
600	220.052	88.75	220.052
660	242.048	88.864	242.048
720	264.044	88.958	264.044
780	286.04	89.038	286.04
840	308.037	89.107	308.037
900	330.035	89.167	330.035
960	352.033	89.219	352.033
1020	374.031	89.265	374.031
1080	396.029	89.306	396.029
1140	418.028	89.342	418.028
1200	440.026	89.375	440.026



SuperHarm Benchmarking - CAPACITOR Modeling Equations

Electrotek Concepts - 6/9/98, TEG

This document illustrates the equations that define the operation of the CAPACITOR model.

CAPACITOR Model Verification:

$Xs := 0.1$ $Rs := 0$

$MVAr := 1.2$ $kV := 13.8$ $f := 60$

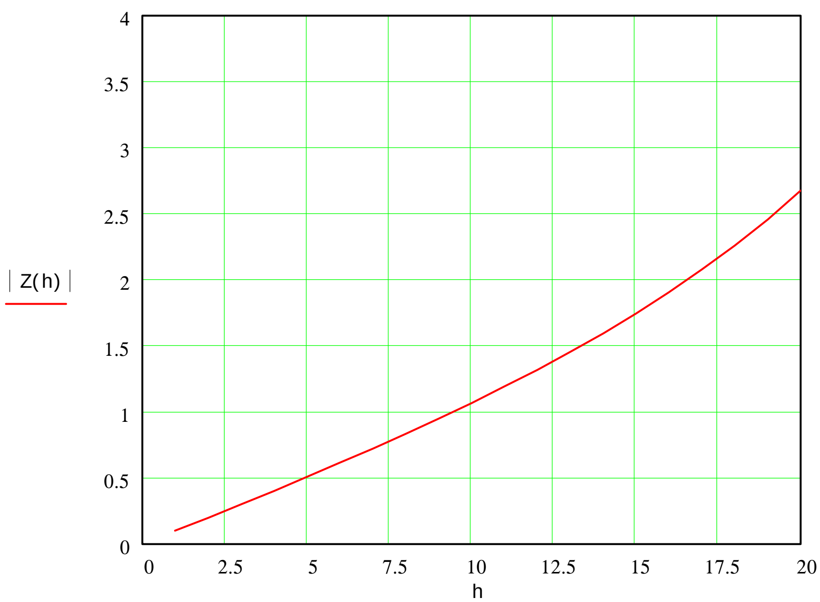
Cases 3a, 3b, 3c, 3d, & 3e:

$Xc := \frac{kV^2 \cdot -1}{MVAr}$ $Xc = -158.7$ $Ls := \frac{Xs}{2 \cdot \pi \cdot 60}$ $Ls = 2.653 \bullet 10^{-4}$ at 60 Hz:

$Xs(h) := j \cdot 2 \cdot \pi \cdot h \cdot f \cdot Ls$ $C := \frac{-1}{Xc \cdot 2 \cdot \pi \cdot f}$ $C = 1.671 \bullet 10^{-5}$ $Xc(h) := \frac{1}{i \cdot 2 \pi \cdot h \cdot f \cdot C}$

$h := 1..20$ $Z(h) := \frac{Xc(h) \cdot Xs(h)}{Xc(h) + Xs(h)}$

h	$\lvert Z(h) \rvert$	$(\arg(Z(h))) \cdot \frac{180}{\pi}$
1	0.100063	90
2	0.200505	90
3	0.301711	90
4	0.404074	90
5	0.508003	90
6	0.613926	90
7	0.722302	90
8	0.833618	90
9	0.948406	90
10	1.067249	90
11	1.190791	90
12	1.319751	90
13	1.454937	90
14	1.597268	90
15	1.747797	90
16	1.907739	90
17	2.078505	90
18	2.261758	90
19	2.459462	90
20	2.673968	90



SuperHarm Benchmarking - CAPACITOR Modeling Equations

Electrotek Concepts - 6/9/98, TEG

This document illustrates the equations that define the operation of the CAPACITOR model.

CAPACITOR Model Verification:

$Xs := 0.1$ $\quad$ $Rs := 0$

$MVAr := 1.2$ $\quad$ $kV := 13.8$ $\quad$ $f := 60$

Cases 3f (delta connected bank):

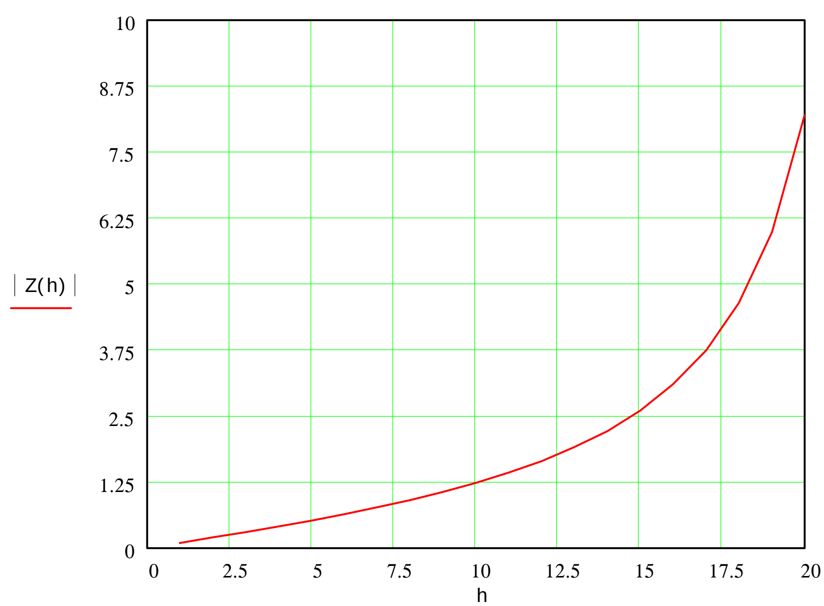
$$Xc := \frac{\dfrac{kV^2 \cdot -1}{MVAr}}{\left(\dfrac{1}{3}\right)} \qquad Xc = -52.9 \qquad Ls := \frac{Xs}{2\cdot\pi\cdot 60} \qquad Ls = 2.653\cdot 10^{-4} \qquad \text{at 60 Hz:}$$

$$C := \frac{-1}{Xc\cdot 2\cdot\pi\cdot f} \qquad C = 5.014\cdot 10^{-5} \qquad Xc(h) := \frac{1}{i\cdot 2\,\pi\cdot h\cdot f\cdot C}$$

$$Xs(h) := j\cdot 2\cdot\pi\cdot h\cdot f\cdot Ls$$

$$h := 1..20 \qquad Z(h) := \frac{Xc(h)\cdot Xs(h)}{Xc(h) + Xs(h)}$$

h	$\lvert Z(h) \rvert$	$(\arg(Z(h)))\cdot\frac{180}{\pi}$
1	0.100189	90
2	0.201524	90
3	0.305192	90
4	0.412476	90
5	0.524802	90
6	0.643813	90
7	0.771458	90
8	0.910108	90
9	1.062723	90
10	1.2331	90
11	1.426225	90
12	1.648831	90
13	1.910278	90
14	2.224024	90
15	2.610197	90
16	3.100366	90
17	3.747083	90
18	4.644878	90
19	5.982738	90
20	8.20155	90



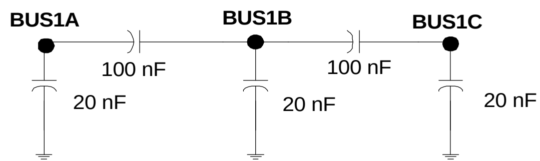
SuperHarm Benchmarking - CMATRIX Modeling Equations

Electrotek Concepts - 6/9/98, TEG

This document illustrates the equations that define the operation of the CMATRIX model.

CMATRIX Model Verification:

$Xs := 1.0 \qquad Rs := 0 \qquad f := 60$



Cases 4d:

$C1 := \left[\frac{(20 \cdot 100)}{(20 + 100)} + 20\right] \qquad C1 = 36.667$

$C2 := \left[\frac{(C1 \cdot 100)}{(C1 + 100)}\right] \qquad C2 = 26.829$

$Ls := \frac{Xs}{2 \cdot \pi \cdot 60} \qquad Ls = 2.653 \cdot 10^{-3}$ at 60 Hz:

$Ctotal := (C2 + 20) \cdot 10^{-9} \qquad Ctotal = 4.683 \cdot 10^{-8}$

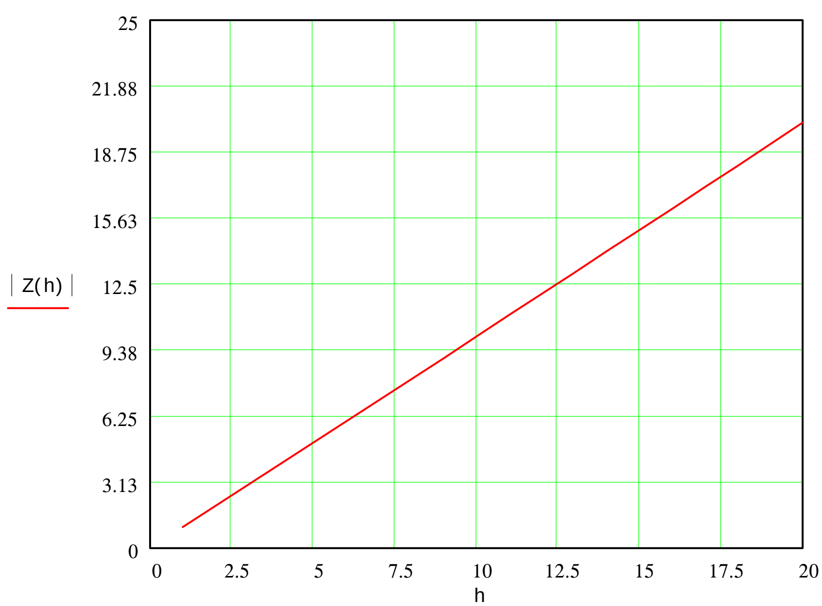
$Xc(h) := \frac{1}{i \cdot 2 \pi \cdot h \cdot f \cdot Ctotal}$

$Xs(h) := j \cdot 2 \cdot \pi \cdot h \cdot f \cdot Ls$

$h := 1 .. 20$

$Z(h) := \frac{Xc(h) \cdot Xs(h)}{Xc(h) + Xs(h)}$

h	$\lvert Z(h) \rvert$	$(\arg(Z(h))) \cdot \frac{180}{\pi}$
1	1.00002	90
2	2.00014	90
3	3.00048	90
4	4.00113	90
5	5.00221	90
6	6.00382	90
7	7.00606	90
8	8.00905	90
9	9.01289	90
10	10.01769	90
11	11.02355	90
12	12.03058	90
13	13.0389	90
14	14.04861	90
15	15.05982	90
16	16.07264	90
17	17.08718	90
18	18.10355	90
19	19.12187	90
20	20.14224	90



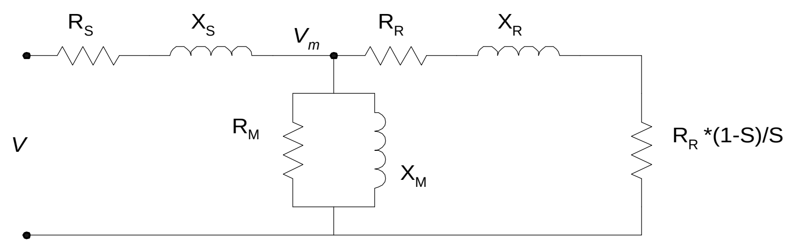
SuperHarm Benchmarking - INDUCTIONMOTOR Modeling Equations

Electrotek Concepts - 6/28/98, EWG/TEG

This document illustrates the equations that define the operation of the INDUCTIONMOTOR model.

INDUCTIONMOTOR Model Verification:

Cases 5a, 5b, 5c, 5d, & 5e :



Note that Rr + Rr*(1-S)/S is equal to Rr/S

Specified Parameters:

$HP := 100$ $V := 277$ $dPF := 0.75$ $f := 60$

! single phase model test

VSOURCE NAME=SRC BUS=VSRC MAG=277

INDUCTIONMOTOR NAME=M1 HP=100 KV=0.277 FROM=VSRC TO=NODE DF = 0.75

Default Parameters:

$Eff := 0.90$ $Load := 1.00$ $s := 0.02$ $p := 2$

$$Zbase := \frac{V^2}{\frac{(HP \cdot 745.6)}{(Eff \cdot dPF)}} \qquad Zbase = 0.69464$$

$$Speed := \frac{120 \cdot f}{p} \qquad Speed = 3600$$

$$\omega s := 2 \cdot \pi \cdot 60 \qquad \omega s = 376.99112$$

$$Speedm := (1 - s) \cdot Speed \qquad Speedm = 3528$$

$$\omega r := \frac{Speedm \cdot \pi}{30} \qquad \omega r = 369.4513$$

$$Pout := HP \cdot 745.6 \qquad Pout = 74560$$

$$slip := \frac{(\omega s - \omega r)}{\omega s} \qquad slip = 0.02$$

$$Pin := \frac{Pout}{Eff} \qquad Pin = 82844.44444$$

$$Sm := \frac{Pin}{dPF} \qquad Sm = 110459.25926$$

$$Qin := \sqrt{\left(Sm^2 - Pin^2\right)} \qquad Qin = 73061.9325$$

$$I := \frac{Sm}{V} \qquad I = 398.76989$$

Reference:
"Studies on Modeling of Harmonic Impedance for Induction Motor",
Zhang Jing, He Fengreng, 2nd ICHPS Conference / October 6-8, 1986 / Pages 134-141

Assumptions:
stator impedance is equal to rotor impedance (Xs = Xr).
exciting impedance (Xm) is 35 times greater that the stator impedance.
exciting current is 30% of stator (rotor = 70% of stator).
core loss of the motor is 3% of nameplate rating.
stator X/R ration = 4

```
 !     <SuperHarm.INF for Case_5a>
 !     Rs,Xs=    0.0247553, j   0.0990213
 !     Rr,Xr=    0.0152459, j   0.0990213
 !     Rm,Xm=       34.303, j     1.26478

 !     Vt   =       0.277  kV L-G         Pout =       74.560  kW
 !     I1   =     398.770  Amps           HPout=      100.000  HP
 !     kVA  =     110.459  kVA            Slip =        0.020
 !     Pf   =       0.750                 RPM  =     3528.000  RPM
 !     Pin  =      82.844  kW             Ploss=     8284.444 Watts
 !     Qin  =      73.062  kVAr           Eff  =       90.000  %
```

$$Xs := \frac{\left(Qin \cdot V^2\right)}{4.64 \cdot \left(Pin^2 + Qin^2\right)} \qquad Xs = 0.09902 \qquad Xr := Xs \qquad Rs := \frac{Xs}{4} \qquad Rs = 0.02476$$

$$Rm := \frac{V^2}{(0.03 \cdot Pin \cdot Eff)} \qquad Rm = 34.303$$

All of the parameters except for the total rotor side resistance is known at this point. This resistance consists of two components, the rotor winding resistance Rr and a resistive component related to the slip. If we can calculate this total rotor side resistance, we can calculate Rr since we know the slip. We do this calculation for full rated load. If the system is operating at less than rated load, we re-calculate the rotor resistance which will give us a new slip.

Since we know the total input watts and vars, we can calculate the total admittance of the motor.

$$Ytm := \frac{Pin}{V^2} - \iota \cdot \frac{Qin}{V^2} \qquad Ytm = 1.0797 - 0.95221\iota \qquad |Ytm| = 1.4396 \qquad \arg(Ytm) \cdot \frac{180}{\pi} = -41.40962$$

$$Zt := \frac{1}{Ytm} \qquad Zt = 0.52098 + 0.45946\iota \qquad |Zt| = 0.6946 \qquad \arg(Zt) \cdot \frac{180}{\pi} = 41.40962$$

Now calculate the admittance of only the magnetizing and rotor components of the motor. This is done simply by subtracting out the stator impedance.

$Zt := Zt - (Rs + \imath \cdot Xs)$ $\quad Zt = 0.49622 + 0.36044\imath$ $\quad |Zt| = 0.6133$ $\quad arg(Zt)\cdot\frac{180}{\pi} = 35.99328$

$Ytm := \frac{1}{Zt}$ $\quad Ytm = 1.319208 - 0.958225\imath$ $\quad |Ytm| = 1.63049$ $\quad arg(Ytm)\cdot\frac{180}{\pi} = -35.99328$

We now know that this admittance is equal to the parallel combination of the magnitizing branch resistance (Rm), magnetizing branch reactance (jXm), and the total rotor impedance (Rr/S + jXr).

Solving for Rr:

$$Ytm = \frac{1}{Rm} + \frac{1}{j\cdot Xm} + \frac{1}{\frac{Rr}{S} + j\cdot Xr}$$

$$Ytm = \frac{1}{Rm} + \frac{1}{j\cdot Xm} + \frac{\frac{Rr}{S} - j\cdot Xr}{\left(\frac{Rr}{S} + j\cdot Xr\right)\cdot\left(\frac{Rr}{S} - j\cdot Xr\right)}$$

$$Ytm = \frac{1}{Rm} + \frac{1}{j\cdot Xm} + \frac{\frac{Rr}{S} - j\cdot Xr}{\frac{Rr^2}{S^2} + Xr^2}$$

Since we know the slip S, the equation may be simplified by substituting Rx for Rr/S:

$$Ytm = \frac{1}{Rm} + \frac{1}{j\cdot Xm} + \frac{Rx - j\cdot Xr}{Rx^2 + Xr^2}$$

Now separate out real and imaginary components:

$$Re(Ytm) + j\cdot Im(Ytm) = \left(\frac{1}{Rm} + \frac{Rx}{Rx^2 + Xr^2}\right) - \frac{j}{Xm} - \frac{j\cdot Xr}{Rx^2 + Xr^2}$$

We now solve the real and imaginary parts of the equation separately.

$$Re(Ytm) = \frac{1}{Rm} + \frac{Rx}{Rx^2 + Xr^2} \qquad Im(Ytm) = -1\cdot\left(\frac{1}{Xm} + \frac{Xr}{Rx^2 + Xr^2}\right)$$

Ok, now we need to solve this equation for Rx, so we re-arrange again. For simplicity, we substitute Re(Ytm)-1/Rm with K1 - we know all of these values, so it is a constant.

$K1 := Re(Ytm) - \frac{1}{Rm}$ $\qquad K1 = 1.29006$

$$K1 = \frac{Rx}{Rx^2 + Xr^2}$$

$$0 = K1 \cdot Rx^2 - Rx + K1 \cdot Xr^2$$

This equation is easily solved using the classic quadratic equation which has two solutions (roots R1 and R2)

$$R1(A, B, C) := \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad R2(A, B, C) := \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$

$$r1 := R1\left(K1, -1, K1 \cdot Xr^2\right) \qquad r1 = 0.7623$$

$$r2 := R2\left(K1, -1, K1 \cdot Xr^2\right) \qquad r2 = -0.37472$$

The positive root is chosen.

$$Rx := r1$$

$$Rr := Rx \cdot slip \qquad Rr = 0.01524594$$

Now we calculate the magnitizing reactance. We can find it by solving the imaginary portion of the equation used to find Rr.

$$Im(Ytm) = -1 \cdot \left(\frac{1}{Xm} + \frac{Xr}{Rx^2 + Xr^2}\right)$$

$$\frac{-1}{Xm} = Im(Ytm) + \frac{Xr}{Rx^2 + Xr^2}$$

$$Xm := \frac{-1}{Im(Ytm) + \frac{Xr}{Rx^2 + Xr^2}} \qquad Xm = 1.26478484$$

Summarizing the key model components at full load:

$Rs = 0.02476$ $Xs = 0.09902$ $Rr = 0.01525$ $Xr = 0.09902$ $Rm = 34.30302$ $Xm = 1.26478$

The preceeding calculations were for full load. If we specified that our output mechanical load was less than full load, then all of the key model parameters will be the same except for the effective rotor resistance and hence the slip. We will now re-calculate the parameters for partial load. The approach entails calculating the Thevenin equivalent source as seen by the mechanical load equivalent R (Rr*(1-s)/s).
We then use the same approach as used previously to calculate the load equivalent.

The Thevenin impedance is the parallel combination of the magnitizing impedance and the stator impedance plus the rotor impedance.

$$Zmag := \frac{1}{\frac{1}{Rm} + \frac{1}{\iota \cdot Xm}}$$

$$Zthev := \frac{1}{\frac{1}{Zmag} + \frac{1}{Rs + \iota \cdot Xs}} + Rr + \iota \cdot Xr \qquad Zthev = 0.03676 + 0.19112\iota$$

The Thevenin voltage is the voltage present across the magnetizing branch when the rotor circuit is open. This is a voltage divider formed by the stator impedance and the magnetizing impedance. The voltage divider formula is used to calculate the equivalent.

$$Vthev := V \cdot \frac{Zmag}{Zmag + Rs + \iota \cdot Xs} \qquad Vthev = 256.6548 + 3.96897\iota$$

Now, we have to write an equation in terms of the known quantities and the unknown mechanical load equivalent resistance. Our circuit now consists of the Thevenin voltage in series with the Thevenin equivalent impedance which then is in series with the mechanical load equivalent resistance Req to complete the circuit. We are given the output power, so we know that the equivalent resistance is related to this resistance by the square of the current through it. We use this to write our equation.

$$Pout = Iout^2 \cdot Req \qquad Iout = \frac{Vthev}{Zthev + Req}$$

$$Pout = \frac{Vthev^2}{(Zthev + Req)^2} \cdot Req$$

$$Pout = \frac{Vthev^2 \cdot Req}{Zthev^2 + 2 \cdot Zthev \cdot Req + Req^2}$$

$$Pout \cdot Zthev^2 + 2 \cdot Pout \cdot Zthev \cdot Req + Pout \cdot Req^2 = Vthev^2 \cdot Req$$

$$Zthev^2 + 2 \cdot Zthev \cdot Req + Req^2 = \frac{Vthev^2 \cdot Req}{Pout}$$

We need to break this equation up into real and imaginary components, and equate the real components on either side of the equation.

$$Zrthev^2 + 2\iota \cdot Zrthev \cdot Zithev + Zithev^2 + 2 \cdot Req \cdot Zrthev + \iota \cdot Zithev + Req^2 = Req \cdot \frac{\left(Vrthev^2 + 2\iota \cdot Vrthev \cdot Vithev^2 + Vithev^2\right)}{Pout}$$

Equate the real components

$$Zrthev^2 + Zithev^2 + 2 \cdot Req \cdot Zrthev + Req^2 = Req \cdot \frac{\left(Vrthev^2 + Vithev^2\right)}{Pout}$$

$$0 = Req^2 + \left[2 \cdot Zrthev - \frac{\left(Vrthev^2 + Vithev^2\right)}{Pout}\right] \cdot Req + Zrthev^2 + Zithev^2$$

Set the output power based on the requested per-unit mechanical load

$Pout := Pout \cdot Load$ $\quad Load = 1$ $\quad Pout = 74560$

Solve for the positive root which is the load equivalent.

$$r1 := R1\left[1, 2 \cdot Re(Zthev) - \frac{Re(Vthev)^2 + Im(Vthev)^2}{Pout}, \left(Re(Zthev)^2 + Im(Zthev)^2\right)\right]$$

$Req := r1$ $\quad Req = 0.76033$

Calculate the new slip for this operating point:

$slip := \frac{Rr}{Req}$ $\quad s := slip$ $\quad slip = 0.02005$ $\quad Rr = 0.01525$

Summarizing the key model components at the requested operating point:

$Rs = 0.02476$ $\quad Xs = 0.09902$

$Rr = 0.01525$ $\quad Xr = 0.09902$

$Rm = 34.30302$ $\quad Xm = 1.26478$

Re-calculate the impedance at fundamental frequency of the motor:

$$Zm := Rs + \iota \cdot Xs + \frac{1}{\frac{1}{Rm} + \frac{1}{\iota \cdot Xm} + \frac{1}{\frac{Rr}{slip} + \iota \cdot Xr}}$$ $\quad Zm = 0.5203 + 0.45841\iota$

$Ym := \frac{1}{Zm}$

$Pin := V^2 \cdot Re(Ym)$ $\quad Qin := -V^2 \cdot Im(Ym)$

$Sm := \sqrt{Pin^2 + Qin^2}$ $\quad dDF := \frac{Pin}{Sm}$

$Eff := \frac{Pout}{Pin}$ $\quad Speedm := (1 - slip) \cdot Speed$

$Pin = 83023.91757$ $\quad Eff = 0.9$

$Qin = 73148.18428$ $\quad Speedm = 3527.81416$

```
!      <SuperHarm.INF for Case_5a>
!      Rs,Xs=     0.0247553, j   0.0990213 ohms
!      Rr,Xr=     0.0152459, j   0.0990213 ohms
!      Rm,Xm=        34.303, j     1.26478 ohms
```

BRANCH NAME=ZSRC
FROM=VSRC TO=NODE r = 1.0

$Zs := 1.0 + \iota \cdot 0.000001$

$|Zs| = 1 \quad arg(Zs)\cdot\frac{180}{\pi} = 0.00006$

$\omega s = 376.99112 \quad \omega r := \frac{Speedm\cdot\pi}{30} \quad \omega r = 369.43184 \quad slip := \frac{(\omega s - \omega r)}{\omega s} \quad slip = 0.02005$

$\omega s(h) := \omega s\cdot h \quad h := 1 \quad \omega s(1) = 376.99112$

$Rmm := 34.303 \quad Lmm := \frac{1.26478}{\omega s(1)} \quad Lmm = 0.00335493 \quad Xmm(h) := 2\cdot\pi\cdot f\cdot h\cdot Lmm \quad Xmm(1) = 1.26478$

$Rr := 0.0152459 \quad Lr := \frac{0.0990213}{\omega s(1)} \quad Lr = 0.00026266 \quad Xr(h) := 2\cdot\pi\cdot f\cdot h\cdot Lr \quad Xr(1) = 0.099021$

$Rs := 0.0247553 \quad Ls := Lr \quad Ls = 0.00026266 \quad Xs(h) := 2\cdot\pi\cdot f\cdot h\cdot Ls \quad Xs(1) = 0.099021$

$Slip(h) := \frac{(\omega s(h) - \omega r)}{\omega s(h)} \quad Slip(1) = 0.02005$

$Rrp(h) := Rr\cdot\frac{1 - Slip(h)}{Slip(h)} \quad Rrp(1) = 0.74509 \quad Rrt(h) := Rrp(h) + Rr$

$Rrt(1) = 0.76033$

$Yr(h) := \frac{1}{Rrp(h) + \iota\cdot Xr(h)} \quad Yr(1) = 1.31883 - 0.17527\iota \quad Zr(h) := \frac{1}{Yr(h)}$

$Ym(h) := \frac{1}{Rmm + \iota\cdot 0} + \frac{1}{0 + i\cdot Xmm(h)} \quad Ym(1) = 0.02915 - 0.79065\iota \quad Zm(h) := \frac{1}{Ym(h)}$

$Zrm(h) := \frac{(Zr(h)\cdot Zm(h))}{Zr(h) + Zm(h)}$

$Zeq(h) := Rs + \iota\cdot Xs(h) + Zrm(h) \quad Zeq(1) = 0.51492 + 0.45026\iota \quad |Zeq(1)| = 0.68401$

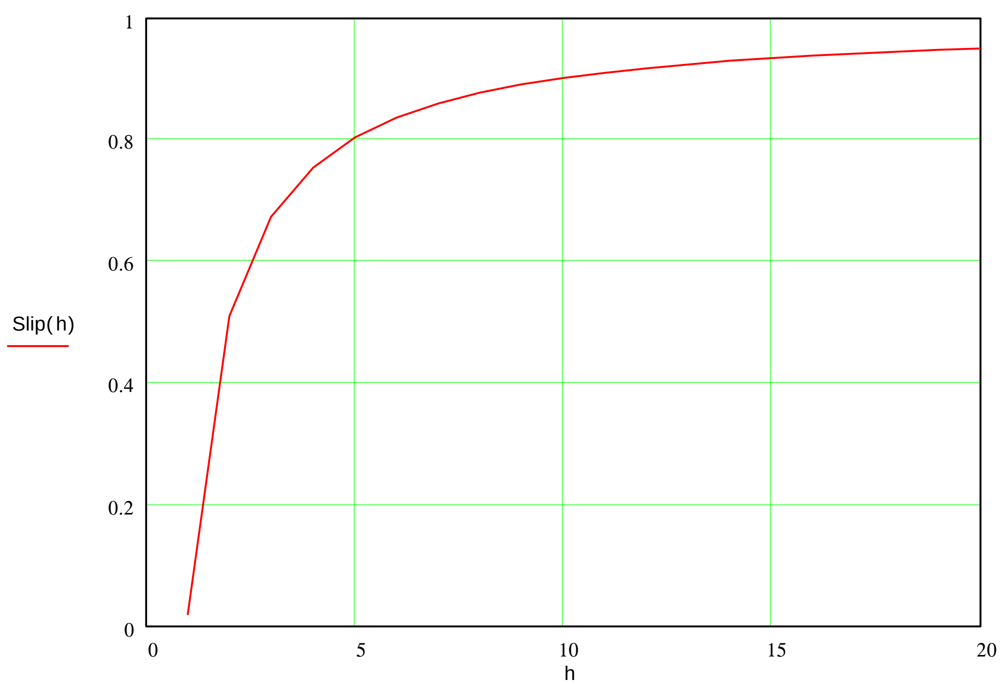
$Zt(h) := \frac{Zs\cdot Zeq(h)}{Zs + Zeq(h)} \quad Zt(1) = 0.393477 + 0.180268\iota$

$|Zt(1)| = 0.432806 \quad arg(Zt(1))\cdot\frac{180}{\pi} = 24.61446$

$h := 1, 2 .. 20$

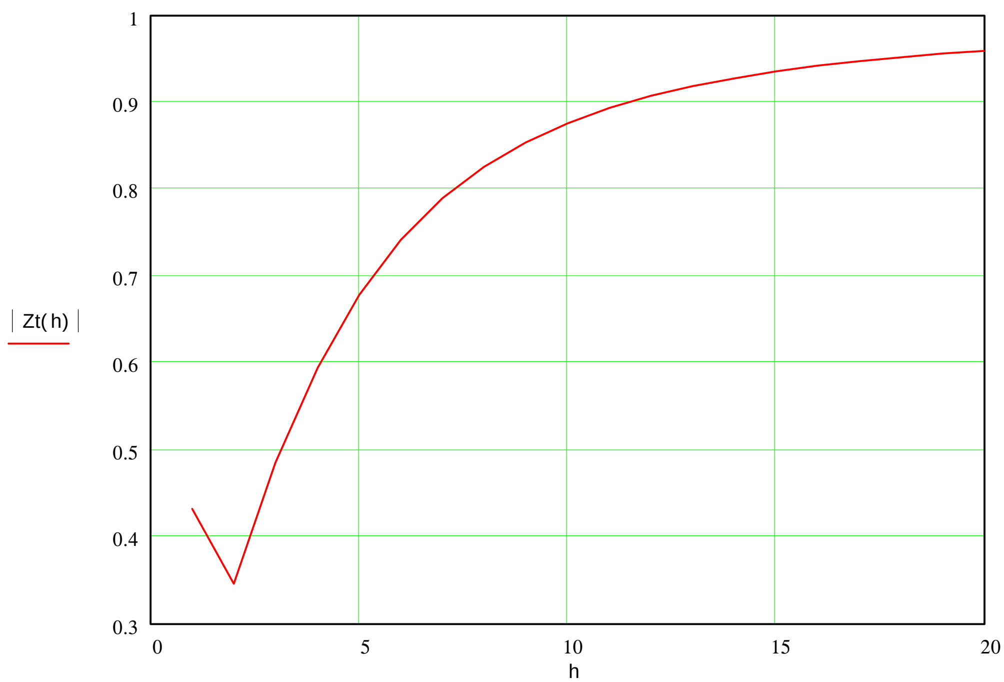
h	$Slip(h)$	$\frac{1 - Slip(h)}{Slip(h)}$	$Rrp(h)$
1	0.02005	48.8713	0.7450866
2	0.51003	0.9607	0.0146465
3	0.67335	0.4851	0.0073959
4	0.75501	0.3245	0.004947
5	0.80401	0.2438	0.0037164
6	0.83668	0.1952	0.0029761
7	0.86001	0.1628	0.0024817
8	0.87751	0.1396	0.0021282
9	0.89112	0.1222	0.0018629
10	0.90201	0.1086	0.0016563
11	0.91091	0.0978	0.001491
12	0.91834	0.0889	0.0013557
13	0.92462	0.0815	0.0012429
14	0.93	0.0753	0.0011475
15	0.93467	0.0699	0.0010656
16	0.93875	0.0652	0.0009947
17	0.94236	0.0612	0.0009326
18	0.94556	0.0576	0.0008778
19	0.94842	0.0544	0.0008291
20	0.951	0.0515	0.0007855

```
! <Case_5e>
branch  name = cr2 from = noder  freqmult = 1
!
!          Freq(Hz) R(ohms) X(ohms)
!          ------------------------
 table=  {
          {   60.0, 0.7470491, 0.0},
          {  120.0, 0.0146480, 0.0},
          {  180.0, 0.0073965, 0.0},
          {  240.0, 0.0049473, 0.0},
          {  300.0, 0.0037167, 0.0},
          {  360.0, 0.0029763, 0.0},
          {  420.0, 0.0024819, 0.0},
          {  480.0, 0.0021283, 0.0},
          {  540.0, 0.0018626, 0.0},
          {  600.0, 0.0016564, 0.0},
          {  660.0, 0.0014911, 0.0},
          {  720.0, 0.0013558, 0.0},
          {  780.0, 0.0012430, 0.0},
          {  840.0, 0.0011475, 0.0},
          {  900.0, 0.0010657, 0.0},
          {  960.0, 0.0009947, 0.0},
          { 1020.0, 0.0009326, 0.0},
          { 1080.0, 0.0008778, 0.0},
          { 1140.0, 0.0008291, 0.0},
          { 1200.0, 0.0007855, 0.0}
          }
```



$h := 1, 2 .. 20$

h	$\|Zeq(h)\|$	$Zt(h)$	$\|Zt(h)\|$	$\arg(Zt(h)) \cdot \frac{180}{\pi}$
1	0.684013	0.3935 + 0.1803i	0.432806	24.61446
2	0.383554	0.1515 + 0.3118i	0.346717	64.08395
3	0.57342	0.2595 + 0.4102i	0.485415	57.68225
4	0.763993	0.3738 + 0.4627i	0.594845	51.06662
5	0.954708	0.4776 + 0.482i	0.67854	45.25752
6	1.145465	0.5654 + 0.4802i	0.741807	40.33694
7	1.336233	0.6372 + 0.4665i	0.789665	36.20764
8	1.526995	0.6949 + 0.4468i	0.826147	32.73954
9	1.717743	0.7412 + 0.4247i	0.854275	29.81128
10	1.908471	0.7785 + 0.4022i	0.876243	27.3208
11	2.099173	0.8087 + 0.3803i	0.893626	25.18577
12	2.289845	0.8333 + 0.3596i	0.907558	23.34078
13	2.480484	0.8535 + 0.3403i	0.918859	21.73411
14	2.671086	0.8703 + 0.3224i	0.928131	20.32479
15	2.861647	0.8844 + 0.3059i	0.935818	19.08018
16	3.052165	0.8963 + 0.2908i	0.942252	17.97412
17	3.242636	0.9063 + 0.2768i	0.947685	16.98548
18	3.433057	0.915 + 0.264i	0.952311	16.09709
19	3.623425	0.9224 + 0.2523i	0.956279	15.29484
20	3.813737	0.9289 + 0.2414i	0.959706	14.56709



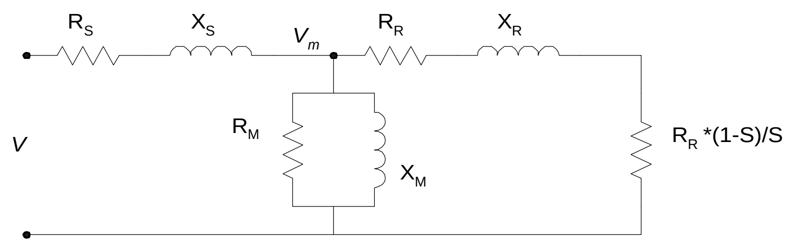
SuperHarm Benchmarking - INDUCTIONMOTOR Modeling Equations

Electrotek Concepts - 10/8/98, EWG/TEG

This document illustrates the equations that define the operation of the INDUCTIONMOTOR model.

INDUCTIONMOTOR **(NEW MODEL)** Model Verification:

Cases 5m thru 5r:



Note that Rr + Rr*(1-S)/S is equal to Rr/S

```
INDUCTIONMOTOR
NAME=MOTOR HP=300
DF=0.75 %EFF=90.0 KV=0.480
POLES=2  %LOAD=100
RPM=3528 FILE=Case5n.inf
```

Specified Parameters:

$HP := 300$ $\quad V := \frac{480}{\sqrt{3}}$ $\quad dPF := 0.75$ $\quad f := 60$

Default Parameters:

$Eff := 0.90$ $\quad Load := 1.00$ $\quad s := 0.02$ $\quad p := 2$

$$Zbase := \frac{V^2}{\left(\frac{HP \cdot 745.6}{3 \cdot Eff \cdot dPF}\right)} \qquad Zbase = 0.69527897$$

$$Speed := \frac{120 \cdot f}{p} \qquad Speed = 3600 \qquad \omega s := 2 \cdot \pi \cdot 60 \qquad \omega s = 376.99111843$$

$$Speedm := (1 - s) \cdot Speed \qquad Speedm = 3528 \qquad \omega r := \frac{Speedm \cdot \pi}{30} \qquad \omega r = 369.45129606$$

$$Pout3 := HP \cdot 745.6 \qquad Pout3 = 223680 \qquad Pout := \frac{Pout3}{3}$$

$$Pin := \frac{Pout}{Eff} \qquad Pin = 82844.44444444 \qquad slip := \frac{(\omega s - \omega r)}{\omega s} \qquad slip = 0.02$$

$$Sm := \frac{Pin}{dPF} \qquad Sm = 110459.25925926$$

$$Qin := \sqrt{\left(Sm^2 - Pin^2\right)} \qquad Qin = 73061.9325011$$

$$Sin := Pin + \iota \cdot Qin \qquad Sin = 82844.44444444 + 73061.9325011\iota \qquad |Sin| = 110459.25925926$$

$$Iin := \overline{\frac{Sin}{V}}$$ $Iin = 298.93913938 - 263.63953998i$ $|Iin| = 398.58551917$

$$I := \frac{Sm}{V}$$ $I = 398.58551917$

Specify locked rotor kva/hp via NEMA code
Code H = 6.7 kva/HP - upper value in NEMA table used

$kvaPerHp := 6.7$

Specify locked rotor power factor. A common default is 0.20. Specifying the efficiency, full load running power factor, and the locked rotor power factor actually over specifies the problem. These three parameters need to be specified, and then one needs to be selected to be allowed to change interatively until a constriaint is met that validates the model. The constraint is for the calculated rotor and stator resistances to add up to the locked rotor resistance. The locked rotor resistance is specified by the locked rotor code and the locked rotor power factor. In this example, the locked rotor power factor is manually adjusted to ensure that the constraint is met.

$DF_{LR} := 0.264$

$kva_{LR} := kvaPerHp \cdot HP$ $kva_{LR} = 2010$

$$I_{LR} := \frac{kva_{LR} \cdot 1000}{3 \cdot V}$$ $I_{LR} = 2417.65425223$

$$Z_{LR} := \frac{V}{I_{LR}}$$ $Z_{LR} = 0.11462687$ $\frac{Z_{LR}}{Zbase} = 0.16486457$

$kW_{LR} := kva_{LR} \cdot DF_{LR}$ $kW_{LR} = 530.64$

$$R_{LR} := \frac{kW_{LR} \cdot 1000}{3 \cdot I_{LR}^2}$$ $R_{LR} = 0.03026149$ $\frac{R_{LR}}{Zbase} = 0.04352425$

$$X_{LR} := \sqrt{Z_{LR}^2 - R_{LR}^2}$$ $X_{LR} = 0.11056021$ $\frac{X_{LR}}{Zbase} = 0.15901561$

Assumptions:
Locked rotor impedance is equal to stator impedance plus rotor impedance
Stator impedance is equal to rotor impedance (Xs = Xr).
Core loss of the motor is 3% of nameplate rating.
Stator X/R ratio = 4

$StatorXR := 4$ $CoreLoss := 0.01$

$$Xr := \frac{X_{LR}}{2}$$ $Xs := Xr$

$Xr = 0.05528011$ $Xs = 0.05528011$ $\frac{Xs}{Zbase} = 0.07950781$

$Rs := \frac{Xs}{StatorXR}$ $Rs = 0.01382003$ $\frac{Rs}{Zbase} = 0.01987695$

$Zs := Rs + \iota \cdot Xs$ $Zs = 0.01382003 + 0.05528011\iota$ $\frac{|Zs|}{Zbase} = 0.08195477$

Calculate the voltage across the magnitizing/core loss branch:

$Vm := V - Iin \cdot Zs$ $Vm = 258.42276072 - 12.88188184\iota$ $|Vm| = 258.74363014$

Calculate the core loss resistance based on the output power and the core loss percentage assumption:

$Rm := \frac{(|Vm|)^2}{CoreLoss \cdot Pout}$ $Rm = 89.79112948$ $Pout3 \cdot CoreLoss = 2236.8$

Here's the deal: We now know all of the parameters except for the total rotor side resistance. This resistance consists of two components, the rotor winding resistance Rr and a resistive component related to the slip. If we can calculate this total rotor side resistance, we can calculate Rr since we know the slip. We do this calculation for full rated load. If the system is operating at less than rated load, we re-calculate the rotor resistance which will give us a new slip.

Since we know the total input current, power factor and the terminal voltage, we can calculate the total admittance of the motor.

$Zin := \frac{V}{Iin}$ $Zin = 0.52145923 + 0.45988381\iota$ $\frac{|Zin|}{Zbase} = 1$

Now calculate the admittance of only the magnetizing and rotor components of the motor. This is done simply by subtracting out the stator impedance.

$Zt := Zin - (Rs + \iota \cdot Xs)$ $Zt = 0.5076392 + 0.40460371\iota$ $|Zt| = 0.6492$ $\arg(Zt) \cdot \frac{180}{\pi} = 38.55589887$

$Ytm := \frac{1}{Zt}$ $Ytm = 1.204644 - 0.960138\iota$ $|Ytm| = 1.54046505$ $\arg(Ytm) \cdot \frac{180}{\pi} = -38.55589887$

We now know that this admittance is equal to the parallel combination of the magnitizing branch resistance (Rm), magnetizing branch reactance (jXm), and the total rotor impedance (Rr/S + jXr). The trick is to write this equation and solve for Rr.

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{1}{\frac{Rr}{S} + j \cdot Xr}$$

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{\frac{Rr}{S} - j \cdot Xr}{\left(\frac{Rr}{S} + j \cdot Xr\right) \cdot \left(\frac{Rr}{S} - j \cdot Xr\right)}$$

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{\frac{Rr}{S} - j \cdot Xr}{\frac{Rr^2}{S^2} + Xr^2}$$

Since we know the slip S, lets make this equation look simpler by substituting Rx for Rr/S:

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{Rx - j \cdot Xr}{Rx^2 + Xr^2}$$

Now separate out real and imaginary components:

$$Re(Ytm) + j \cdot Im(Ytm) = \left(\frac{1}{Rm} + \frac{Rx}{Rx^2 + Xr^2}\right) - \frac{j}{Xm} - \frac{j \cdot Xr}{Rx^2 + Xr^2}$$

We now solve the real and imaginary parts of the equation separately.

$$Re(Ytm) = \frac{1}{Rm} + \frac{Rx}{Rx^2 + Xr^2} \qquad Im(Ytm) = -1 \cdot \left(\frac{1}{Xm} + \frac{Xr}{Rx^2 + Xr^2}\right)$$

Ok, now we need to solve this equation for Rx, so we re-arrange again. For simplicity, lets substitute Re(Ytm)-1/Rm with K1 - we know all of these values, so it is a constant.

$$K1 := Re(Ytm) - \frac{1}{Rm} \qquad K1 = 1.19351$$

$$K1 = \frac{Rx}{Rx^2 + Xr^2}$$

$$0 = K1 \cdot Rx^2 - Rx + K1 \cdot Xr^2$$

This equation is easily solved using the classic quadratic equation which has two solutions (roots R1 and R2)

$$R1(A, B, C) := \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad R2(A, B, C) := \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$

$$r1 := R1\left(K1, -1, K1 \cdot Xr^2\right) \qquad r1 = 0.83420337$$

$$r2 := R2\left(K1, -1, K1 \cdot Xr^2\right) \qquad r2 = -0.41527006$$

$$Rs = 0.01382003$$

We choose the positive root.

$$Rx := r1$$

$$Rr := Rx \cdot slip \qquad Rr = 0.01668$$

Now we calculate the magnitizing reactance. We can find it by solving the imaginary portion of the equation used to find Rr.

$$\mathrm{Im}(Ytm) = -1 \cdot \left(\frac{1}{Xm} + \frac{Xr}{Rx^2 + Xr^2} \right)$$

$$\frac{-1}{Xm} = \mathrm{Im}(Ytm) + \frac{Xr}{Rx^2 + Xr^2}$$

$$Xm := \frac{-1}{\mathrm{Im}(Ytm) + \frac{Xr}{Rx^2 + Xr^2}} \qquad Xm = 1.13501234$$

Summarizing the key model components at full load:

$Rs = 0.01382003$ $Xs = 0.05528011$ $\frac{2 \cdot Xs}{Zbase} = 0.15901561$

$Rr = 0.01668407$ $Xr = 0.05528011$

$Rm = 89.79112948$ $Xm = 1.13501234$ $\frac{Xm}{Xs} = 20.53202191$

As mentioned above, the problem is over specified when we provide the efficiency, power factor and the locked rotor test power factor. This results in a calculation of Rr that may not be consistent with the locked rotor values. This can be checked by summing Rs and Rr and comparing with the total locked rotor resistance RXL. An iterative procedure can be used to vary the given running PF, efficiency, or locked rotor test PF to generate a value of Rr that meets the Rr + Rs == RXL constraint.

$R_{LR} = 0.03026149$ $Rs + Rr = 0.03050409$

$$Iout := \frac{Vm}{\frac{Rr}{slip} + \iota \cdot Xr} \qquad Iout = 307.41064826 - 35.8132995\iota \qquad |Iout| = 309.48973986$$

$$Pr := (|Iout|)^2 \cdot Rr \cdot 3 \qquad Pr = 4794.19506544$$

Equivalent resistance representing the mechanical load:

$$Req := Rr \cdot \frac{(1 - slip)}{slip} \qquad Req = 0.8175193$$

Power dissipated by the resistor representing the energy converted to mechanical work. It should match the alternate calculation shown below:

$$Pconv := (|Iout|)^2 \cdot Req \cdot 3 \qquad Pconv = 234915.55820643$$

Input power (3 phase):

$$Pin3 := Pin \cdot 3 \qquad Pin3 = 248533.33333333$$

Stator power loss:

$Ps := (|Iin|)^2 \cdot Rs \cdot 3$ $\quad Ps = 6586.78006146 \quad \frac{Ps}{Pin3} = 2.65026022 \circ\%$

Core loss:

$Pm := \frac{(|Vm|)^2}{Rm} \cdot 3$ $\quad Pm = 2236.8 \quad \frac{Pm}{Pin3} = 0.9 \circ\%$

Power transferred across the air gap:

$Pag := Pin3 - Ps - Pm$ $\quad Pag = 239709.75327187 \quad \frac{Pag}{Pin3} = 96.44973978 \circ\%$

Rotor power loss:

$Prcu := Pag \cdot slip$ $\quad Prcu = 4794.19506544 \quad \frac{Prcu}{Pin3} = 1.9289948 \circ\%$

Power converted to mechanical energy:

$Pconv := Pag - Prcu$ $\quad Pconv = 234915.55820643 \quad \frac{Pconv}{Pin3} = 94.52074499 \circ\%$

Power loss due to friction and windage:

$Pfw := Pconv - Pout3$ $\quad Pfw = 11235.55820643 \quad \frac{Pfw}{Pin3} = 4.52074499 \circ\%$

Power at the shaft:

$HP = 300$ $\quad Pout3 = 223680 \quad \frac{Pout3}{Pin3} = 90 \circ\%$

The preceeding calculations were for full load. If we specified that our output mechanical load was less than full load, then all of the key model parameters will be the same except for the effective rotor resistance and hence the slip. We will now re-calculate the parameters for partial load. The approach entails calculating the Thevenin equivalent source as seen by the mechanical load equivalent R (Rr*(1-s)/s). We then use the same approach as used previously to calculate the load equivalent.

The Thevenin impedance is the parallel combination of the magnitizing impedance and the stator impedance plus the rotor impedance.

$$Zmag := \frac{1}{\frac{1}{Rm} + \frac{1}{\iota \cdot Xm}}$$

$$Zthev := \frac{1}{\frac{1}{Zmag} + \frac{1}{Rs + \iota \cdot Xs}} + Rr + \iota \cdot Xr \qquad Zthev = 0.02927788 + 0.10812394\iota$$

The Thevenin voltage is the voltage present across the magnetizing branch when the rotor circuit is open. This is a voltage divider formed by the stator impedance and the magnetizing impedance. The voltage divider formula is used to calculate the equivalent.

$$Vthev := V \cdot \frac{Zmag}{Zmag + Rs + \iota \cdot Xs} \qquad Vthev = 264.18674572 + 2.91184962\iota$$

Now, we have to write an equation in terms of the known quantities and the unknown mechanical load equivalent resistance. Our circuit now consists of the Thevenin voltage in series with the Thevenin equivalent impedance which then is in series with the mechanical load equivalent resistance Req to complete the circuit. We are given the output power, so we know that the equivalent resistance is related to this resistance by the square of the current through it. We use this to write our equation.

$$Pconv = (|Iout|)^2 \cdot Req$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{(|Zthev + Req|)^2}$$

$$(|Zthev + Req|)^2 = (Zrthev + Req)^2 + Zithev^2$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{(Zrthev + Req)^2 + Zithev^2}$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{Zrthev^2 + 2 \cdot Zrthev \cdot Req + Req^2 + Zithev^2}$$

$$(|Zthev|)^2 = Zrthev^2 + Zithev^2$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{(|Zthev|)^2 + 2 \cdot Zrthev \cdot Req + Req^2}$$

$$Pconv = \frac{(|Vthev|)^2 \cdot Req}{(|Zthev|)^2 + 2 \cdot Zrthev \cdot Req + Req^2}$$

$$Pconv \cdot (|Zthev|)^2 + 2 \cdot Pconv \cdot Zthev \cdot Req + Pconv \cdot Req^2 = (|Vthev|)^2 \cdot Req$$

$$(|Zthev|)^2 + 2 \cdot Zthev \cdot Req + Req^2 = \frac{(|Vthev|)^2 \cdot Req}{Pconv}$$

$$0 = Req^2 + \left[2 \cdot Zrthev - \frac{(|Vthev|)^2}{Pout}\right] \cdot Req + (|Zthev|)^2$$

Set the mechanical output power based on the requested per-unit mechanical load. Assume that the friction and windage loss component remains constant.

$Load = 1$

$HP_{new} := HP \cdot Load$ $\quad$ $HP_{new} = 300$

$Pout3_{new} := HP_{new} \cdot 745.6$ $\quad$ $Pout3_{new} = 223680$

$Pconv_{new} := Pout3_{new} + Pfw \qquad Pconv_{new} = 234915.55820643$

Solve for the positive root which is the load equivalent resistance.

$$r1 := R1\left[1, 2 \cdot Re(Zthev) - \frac{(|Vthev|)^2}{\frac{Pconv_{new}}{3}}, (|Zthev|)^2\right]$$

$Req := r1 \qquad Req = 0.8175193$

Calculate the new slip for this operating point:

$slip_{new} := \frac{Rr}{Rr + Req} \qquad slip_{new} = 0.02$

$Pout3_{new} := Pconv_{new} - Pfw \qquad Pout3_{new} = 223680$

$HP_{new} := \frac{Pout3_{new}}{745.6} \qquad HP_{new} = 300$

Re-calculate the impedance at fundamental frequency of the motor:

$$Zm := Rs + \iota \cdot Xs + \frac{1}{\frac{1}{Rm} + \frac{1}{\iota \cdot Xm} + \frac{1}{\frac{Rr}{slip_{new}} + \iota \cdot Xr}} \qquad Zm = 0.52145923 + 0.45988381\iota$$

$Ym := \frac{1}{Zm} \qquad Ym = 1.0787037 - 0.95132725\iota$

$Pin_{new} := V^2 \cdot Re(Ym) \qquad Pin_{new} = 82844.44444444$

$Qin_{new} := -V^2 \cdot Im(Ym) \qquad Qin_{new} = 73061.9325011$

$Pout_{new} := \frac{Pout3_{new}}{3} \qquad Pout_{new} = 74560$

$Sm_{new} := \sqrt{Pin_{new}^2 + Qin_{new}^2}$

$dDF_{new} := \frac{Pin_{new}}{Sm_{new}} \qquad dDF_{new} = 0.75$

$Eff := \frac{Pout_{new}}{Pin_{new}}$

$Eff = 0.9$

$Speedm = 3528$ $Load = 1$

$Speedm_{new} := (1 - slip_{new}) \cdot Speed$

$Speedm_{new} = 3528$

$Speedm := Speedm_{new}$

$Speedm = 3528$

BRANCH NAME=ZSRC FROM=VSRC TO=NODE r = 1.0

$Zs := 1.0 + i \cdot 0.000001$ $|Zs| = 1$ $\arg(Zs) \cdot \frac{180}{\pi} = 0.0000573$

$\omega s = 376.99111843$ $\omega r := \frac{Speedm \cdot \pi}{30}$ $\omega r = 369.45129606$ $slip := \frac{(\omega s - \omega r)}{\omega s}$ $slip = 0.02$

$\omega s(h) := \omega s \cdot h$ $h := 1$ $\omega s(1) = 376.99111843$

$Rmm := Rm$ $Lmm := \frac{Xm}{\omega s(1)}$ $Lmm = 0.00301071$ $Xmm(h) := 2 \cdot \pi \cdot f \cdot h \cdot Lmm$ $Xmm(1) = 1.13501234$

$Rr = 0.01668407$ $Lr := \frac{Xr}{\omega s(1)}$ $Lr = 0.00014664$ $Xr(h) := 2 \cdot \pi \cdot f \cdot h \cdot Lr$ $Xr(1) = 0.05528$

$Rs = 0.01382003$ $Ls := Lr$ $Ls = 0.00014664$ $Xs(h) := 2 \cdot \pi \cdot f \cdot h \cdot Ls$ $Xs(1) = 0.05528$

$Slip(h) := \frac{(\omega s(h) - \omega r)}{\omega s(h)}$ $Slip(1) = 0.02$

$Rrp(h) := Rr \cdot \frac{1 - Slip(h)}{Slip(h)}$ $Rrp(1) = 0.81752$ $Rrt(h) := Rrp(h) + Rr$

$Rrt(1) = 0.83420337$

$Yr(h) := \frac{1}{Rrp(h) + i \cdot Xr(h)}$ $Yr(1) = 1.21764518 - 0.0823363$ $Zr(h) := \frac{1}{Yr(h)}$

$Ym(h) := \frac{1}{Rmm + i \cdot 0} + \frac{1}{0 + i \cdot Xmm(h)}$ $Ym(1) = 0.01113696 - 0.881047$ $Zm(h) := \frac{1}{Ym(h)}$

$Zrm(h) := \frac{(Zr(h) \cdot Zm(h))}{Zr(h) + Zm(h)}$

$Zeq(h) := Rs + \iota \cdot Xs(h) + Zrm(h)$ $\quad Zeq(1) = 0.51782942 + 0.4504312\iota$ $\quad |Zeq(1)| = 0.68632032$

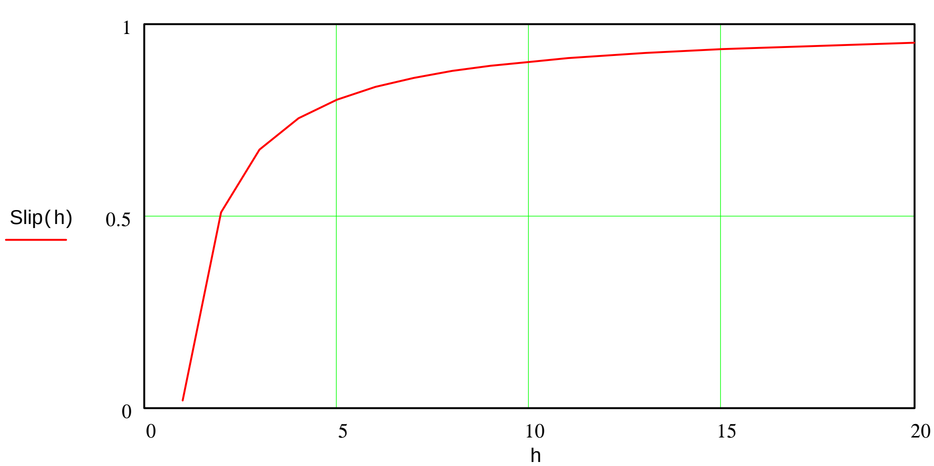
$Zt(h) := \dfrac{Zs \cdot Zeq(h)}{Zs + Zeq(h)}$ $\quad Zt(1) = 0.39449 + 0.179691\iota$

$|Zt(1)| = 0.433487$ $\quad arg(Zt(1)) \cdot \dfrac{180}{\pi} = 24.4894407$

$h := 1, 2 .. 20$

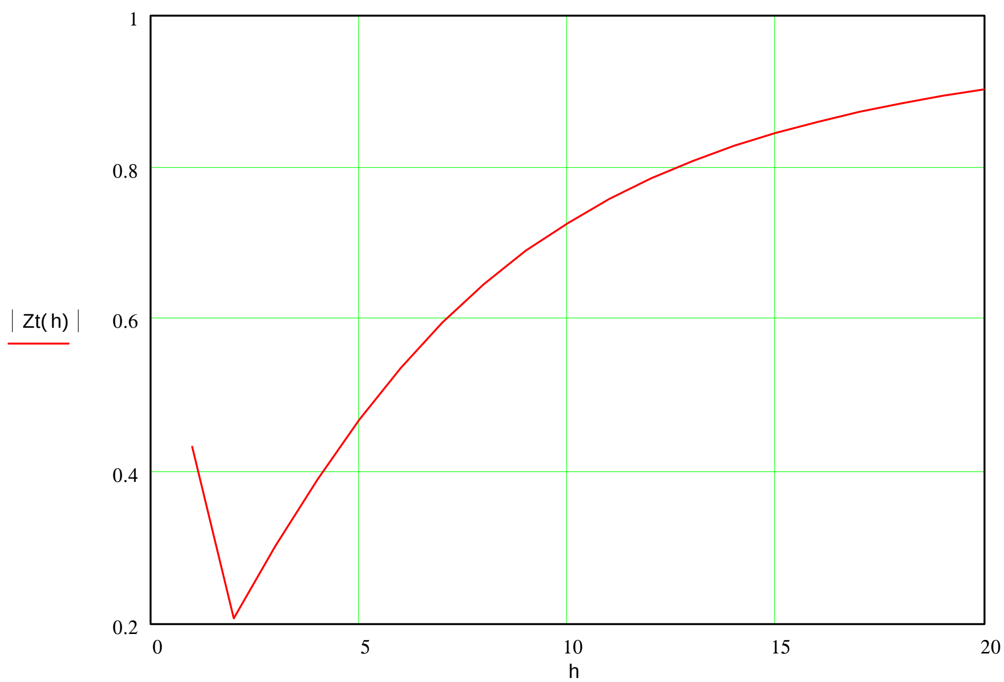
h	Slip(h)	$\dfrac{1 - Slip(h)}{Slip(h)}$	Rrp(h)
1	0.02	49	0.8175193
2	0.51	0.9608	0.0160298
3	0.67333	0.4851	0.0080943
4	0.755	0.3245	0.005414
5	0.804	0.2438	0.0040673
6	0.83667	0.1952	0.003257
7	0.86	0.1628	0.002716
8	0.8775	0.1396	0.0023291
9	0.89111	0.1222	0.0020387
10	0.902	0.1086	0.0018127
11	0.91091	0.0978	0.0016318
12	0.91833	0.0889	0.0014837
13	0.92462	0.0815	0.0013603
14	0.93	0.0753	0.0012558
15	0.93467	0.0699	0.0011662
16	0.93875	0.0652	0.0010886
17	0.94235	0.0612	0.0010206
18	0.94556	0.0576	0.0009607
19	0.94842	0.0544	0.0009073
20	0.951	0.0515	0.0008596

```
! <Case_5e>
branch  name = cr2 from = noder  freqmult = 1
!
!         Freq(Hz) R(ohms) X(ohms)
!         ------------------------
 table= {
         {   60.0, 0.7470491, 0.0},
         {  120.0, 0.0146480, 0.0},
         {  180.0, 0.0073965, 0.0},
         {  240.0, 0.0049473, 0.0},
         {  300.0, 0.0037167, 0.0},
         {  360.0, 0.0029763, 0.0},
         {  420.0, 0.0024819, 0.0},
         {  480.0, 0.0021283, 0.0},
         {  540.0, 0.0018626, 0.0},
         {  600.0, 0.0016564, 0.0},
         {  660.0, 0.0014911, 0.0},
         {  720.0, 0.0013558, 0.0},
         {  780.0, 0.0012430, 0.0},
         {  840.0, 0.0011475, 0.0},
         {  900.0, 0.0010657, 0.0},
         {  960.0, 0.0009947, 0.0},
         { 1020.0, 0.0009326, 0.0},
         { 1080.0, 0.0008778, 0.0},
         { 1140.0, 0.0008291, 0.0},
         { 1200.0, 0.0007855, 0.0}
         }
```



$h := 1, 2 .. 20$

h	$\lvert Zeq(h) \rvert$	$Zt(h)$	$\lvert Zt(h) \rvert$	$arg(Zt(h)) \cdot \frac{180}{\pi}$
1	0.68632	0.3945 + 0.1797i	0.433487	24.4894407
2	0.217923	0.0688 + 0.1956i	0.207356	70.61795562
3	0.324679	0.1105 + 0.2821i	0.302984	68.6134969
4	0.432381	0.1683 + 0.3525i	0.390591	64.48271846
5	0.540253	0.2335 + 0.4064i	0.46873	60.12524789
6	0.648181	0.3009 + 0.445i	0.537187	55.93946764
7	0.756134	0.3668 + 0.4703i	0.596408	52.04939941
8	0.8641	0.4289 + 0.4847i	0.647215	48.49034415
9	0.972073	0.4861 + 0.4905i	0.690598	45.26049516
10	1.080051	0.5378 + 0.4901i	0.727569	42.34122142
11	1.188031	0.584 + 0.4849i	0.75908	39.7067405
12	1.296013	0.625 + 0.4766i	0.785982	37.32913437
13	1.403995	0.6612 + 0.4661i	0.809011	35.1809756
14	1.511978	0.6932 + 0.4543i	0.828791	33.23663382
15	1.61996	0.7214 + 0.4416i	0.845848	31.47283811
16	1.727942	0.7463 + 0.4286i	0.860616	29.86882491
17	1.835924	0.7683 + 0.4155i	0.873458	28.40626449
18	1.943904	0.7878 + 0.4026i	0.884672	27.06907838
19	2.051884	0.805 + 0.3899i	0.894506	25.84321248
20	2.159862	0.8204 + 0.3776i	0.903165	24.71640164



<Case5r.inf>

```
      Induction motor component info for LOAD

                          Vm
     ---[ Rs ]--[ Xs ]-----[ Rr ]--[ Xr ]--+
      +                  |                 |
      |               +--+--+             +++      +-   -+
      |               |     |             | |      |1 - S|
      Vt             +++   +++            | | Rr x|-----|
      |           Rm | |   | | Xm         +++      |  S  |
      |              +++   +++             |       +-   -+
      +               |     |              |
     ----------------+-----+---------------+

      Rs,Xs=    0.0138119, j   0.0552475
      Rr,Xr=    0.0166853, j   0.0552475
      Rm,Xm=      89.7985, j     1.13494

      Vt    =        0.480  kV L-L         Pout =      223.680  kW
      Iin   =      398.586  Amps           HPout=      300.000  HP
      kVA   =      331.378  kVA            Slip =        0.020
      PF    =        0.750                 RPM  =     3528.000  RPM
      Pin   =      248.533  kW             Ploss=    24853.333  Watts
      Qin   =      219.186  kVAr           Eff  =       90.000
   Pstator =     6582.894  Watts          Prot =     4794.273  Watts
     Pconv =      234.919  kW             Pcore=     2236.800  Watts
       Pag =      239.714  kW             Pfw  =    11239.367  Watts
```

Parameter Summary:

Rs = 0.01382003 Xs(1) = 0.05528011

Rr = 0.01668407 Xr(1) = 0.05528011

Rm = 89.79112948 Xm = 1.13501234

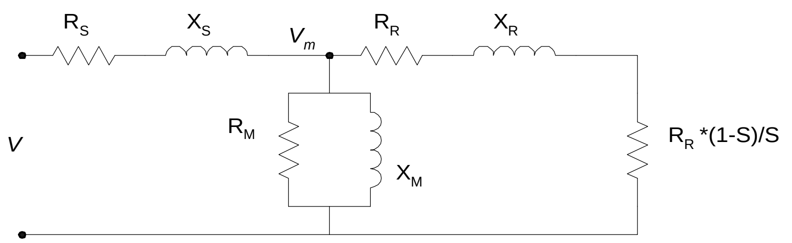
SuperHarm Benchmarking - INDUCTIONMOTOR Modeling Equations

Electrotek Concepts - 10/8/98, EWG/TEG

This document illustrates the equations that define the operation of the INDUCTIONMOTOR model.

INDUCTIONMOTOR **(NEW MODEL)** Model Verification:

Cases 5m thru 5r:



Note that Rr + Rr*(1-S)/S is equal to Rr/S

```
INDUCTIONMOTOR
NAME=MOTOR HP=300
DF=0.75 %EFF=90.0 KV=0.480
POLES=2  %LOAD=100
RPM=3528 FILE=Case5n.inf
```

Specified Parameters:

$HP := 300 \qquad V := \frac{480}{\sqrt{3}} \qquad dPF := 0.75 \qquad f := 60$

Default Parameters:

$Eff := 0.90 \qquad Load := 0.5 \qquad s := 0.02 \qquad p := 2$

$Zbase := \frac{V^2}{\left(\frac{HP \cdot 745.6}{3 \cdot Eff \cdot dPF}\right)} \qquad Zbase = 0.69527897$

$Speed := \frac{120 \cdot f}{p} \qquad Speed = 3600 \qquad \omega s := 2 \cdot \pi \cdot 60 \qquad \omega s = 376.99111843$

$Speedm := (1 - s) \cdot Speed \qquad Speedm = 3528 \qquad \omega r := \frac{Speedm \cdot \pi}{30} \qquad \omega r = 369.45129606$

$Pout3 := HP \cdot 745.6 \qquad Pout3 = 223680 \qquad Pout := \frac{Pout3}{3}$

$Pin := \frac{Pout}{Eff} \qquad Pin = 82844.44444444 \qquad slip := \frac{(\omega s - \omega r)}{\omega s} \qquad slip = 0.02$

$Sm := \frac{Pin}{dPF} \qquad Sm = 110459.25925926$

$Qin := \sqrt{\left(Sm^2 - Pin^2\right)} \qquad Qin = 73061.9325011$

$Sin := Pin + \iota \cdot Qin \qquad Sin = 82844.44444444 + 73061.9325011\iota \qquad |Sin| = 110459.25925926$

$Iin := \overline{\frac{Sin}{V}}$ $Iin = 298.93913938 - 263.63953998i$ $|Iin| = 398.58551917$

$I := \frac{Sm}{V}$ $I = 398.58551917$

Specify locked rotor kva/hp via NEMA code
Code H = 6.7 kva/HP - upper value in NEMA table used

$kvaPerHp := 6.7$

Specify locked rotor power factor. A common default is 0.20. Specifying the efficiency, full load running power factor, and the locked rotor power factor actually over specifies the problem. These three parameters need to be specified, and then one needs to be selected to be allowed to change interatively until a constriaint is met that validates the model. The constraint is for the calculated rotor and stator resistances to add up to the locked rotor resistance. The locked rotor resistance is specified by the locked rotor code and the locked rotor power factor. In this example, the locked rotor power factor is manually adjusted to ensure that the constraint is met.

$DF_{LR} := 0.264$

$kva_{LR} := kvaPerHp \cdot HP$ $kva_{LR} = 2010$

$I_{LR} := \frac{kva_{LR} \cdot 1000}{3 \cdot V}$ $I_{LR} = 2417.65425223$

$Z_{LR} := \frac{V}{I_{LR}}$ $Z_{LR} = 0.11462687$ $\frac{Z_{LR}}{Zbase} = 0.16486457$

$kW_{LR} := kva_{LR} \cdot DF_{LR}$ $kW_{LR} = 530.64$

$R_{LR} := \frac{kW_{LR} \cdot 1000}{3 \cdot I_{LR}^2}$ $R_{LR} = 0.03026149$ $\frac{R_{LR}}{Zbase} = 0.04352425$

$X_{LR} := \sqrt{Z_{LR}^2 - R_{LR}^2}$ $X_{LR} = 0.11056021$ $\frac{X_{LR}}{Zbase} = 0.15901561$

Assumptions:
Locked rotor impedance is equal to stator impedance plus rotor impedance
Stator impedance is equal to rotor impedance (Xs = Xr).
Core loss of the motor is 3% of nameplate rating.
Stator X/R ratio = 4

$StatorXR := 4$ $CoreLoss := 0.01$

$Xr := \frac{X_{LR}}{2}$ $Xs := Xr$

$Xr = 0.05528011$ $Xs = 0.05528011$ $\frac{Xs}{Zbase} = 0.07950781$

$Rs := \frac{Xs}{StatorXR}$ $\quad Rs = 0.01382003$ $\quad \frac{Rs}{Zbase} = 0.01987695$

$Zs := Rs + \iota \cdot Xs$ $\quad Zs = 0.01382003 + 0.05528011\iota$ $\quad \frac{|Zs|}{Zbase} = 0.08195477$

Calculate the voltage across the magnitizing/core loss branch:

$Vm := V - Iin \cdot Zs$ $\quad Vm = 258.42276072 - 12.88188184\iota$ $\quad |Vm| = 258.74363014$

Calculate the core loss resistance based on the output power and the core loss percentage assumption:

$Rm := \frac{(|Vm|)^2}{CoreLoss \cdot Pout}$ $\quad Rm = 89.79112948$ $\quad Pout3 \cdot CoreLoss = 2236.8$

Here's the deal: We now know all of the parameters except for the total rotor side resistance. This resistance consists of two components, the rotor winding resistance Rr and a resistive component related to the slip. If we can calculate this total rotor side resistance, we can calculate Rr since we know the slip. We do this calculation for full rated load. If the system is operating at less than rated load, we re-calculate the rotor resistance which will give us a new slip.

Since we know the total input current, power factor and the terminal voltage, we can calculate the total admittance of the motor.

$Zin := \frac{V}{Iin}$ $\quad Zin = 0.52145923 + 0.45988381\iota$ $\quad \frac{|Zin|}{Zbase} = 1$

Now calculate the admittance of only the magnetizing and rotor components of the motor. This is done simply by subtracting out the stator impedance.

$Zt := Zin - (Rs + \iota \cdot Xs)$ $\quad Zt = 0.5076392 + 0.40460371\iota$ $\quad |Zt| = 0.6492$ $\quad arg(Zt) \cdot \frac{180}{\pi} = 38.55589887$

$Ytm := \frac{1}{Zt}$ $\quad Ytm = 1.204644 - 0.960138\iota$ $\quad |Ytm| = 1.54046505$ $\quad arg(Ytm) \cdot \frac{180}{\pi} = -38.55589887$

We now know that this admittance is equal to the parallel combination of the magnitizing branch resistance (Rm), magnetizing branch reactance (jXm), and the total rotor impedance (Rr/S + jXr). The trick is to write this equation and solve for Rr.

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{1}{\frac{Rr}{S} + j \cdot Xr}$$

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{\frac{Rr}{S} - j \cdot Xr}{\left(\frac{Rr}{S} + j \cdot Xr\right) \cdot \left(\frac{Rr}{S} - j \cdot Xr\right)}$$

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{\frac{Rr}{S} - j \cdot Xr}{\frac{Rr^2}{S^2} + Xr^2}$$

Since we know the slip S, lets make this equation look simpler by substituting Rx for Rr/S:

$$Ytm = \frac{1}{Rm} + \frac{1}{j \cdot Xm} + \frac{Rx - j \cdot Xr}{Rx^2 + Xr^2}$$

Now separate out real and imaginary components:

$$Re(Ytm) + j \cdot Im(Ytm) = \left(\frac{1}{Rm} + \frac{Rx}{Rx^2 + Xr^2}\right) - \frac{j}{Xm} - \frac{j \cdot Xr}{Rx^2 + Xr^2}$$

We now solve the real and imaginary parts of the equation separately.

$$Re(Ytm) = \frac{1}{Rm} + \frac{Rx}{Rx^2 + Xr^2} \qquad Im(Ytm) = -1 \cdot \left(\frac{1}{Xm} + \frac{Xr}{Rx^2 + Xr^2}\right)$$

Ok, now we need to solve this equation for Rx, so we re-arrange again. For simplicity, lets substitute Re(Ytm)-1/Rm with K1 - we know all of these values, so it is a constant.

$$K1 := Re(Ytm) - \frac{1}{Rm} \qquad K1 = 1.19351$$

$$K1 = \frac{Rx}{Rx^2 + Xr^2}$$

$$0 = K1 \cdot Rx^2 - Rx + K1 \cdot Xr^2$$

This equation is easily solved using the classic quadratic equation which has two solutions (roots R1 and R2)

$$R1(A, B, C) := \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad R2(A, B, C) := \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$

$$r1 := R1\left(K1, -1, K1 \cdot Xr^2\right) \qquad r1 = 0.83420337$$

$$r2 := R2\left(K1, -1, K1 \cdot Xr^2\right) \qquad r2 = -0.41527006$$

$$Rs = 0.01382003$$

We choose the positive root.

$$Rx := r1$$

$$Rr := Rx \cdot slip \qquad Rr = 0.01668$$

Now we calculate the magnitizing reactance. We can find it by solving the imaginary portion of the equation used to find Rr.

$$Im(Ytm) = -1 \cdot \left(\frac{1}{Xm} + \frac{Xr}{Rx^2 + Xr^2} \right)$$

$$\frac{-1}{Xm} = Im(Ytm) + \frac{Xr}{Rx^2 + Xr^2}$$

$$Xm := \frac{-1}{Im(Ytm) + \frac{Xr}{Rx^2 + Xr^2}} \qquad Xm = 1.13501234$$

Summarizing the key model components at full load:

$Rs = 0.01382003$	$Xs = 0.05528011$	$\frac{2 \cdot Xs}{Zbase} = 0.15901561$
$Rr = 0.01668407$	$Xr = 0.05528011$	
$Rm = 89.79112948$	$Xm = 1.13501234$	$\frac{Xm}{Xs} = 20.53202191$

As mentioned above, the problem is over specified when we provide the efficiency, power factor and the locked rotor test power factor. This results in a calculation of Rr that may not be consistent with the locked rotor values. This can be checked by summing Rs and Rr and comparing with the total locked rotor resistance RXL. An iterative procedure can be used to vary the given running PF, efficiency, or locked rotor test PF to generate a value of Rr that meets the Rr + Rs == RXL constraint.

$R_{LR} = 0.03026149$ $\qquad Rs + Rr = 0.03050409$

$$Iout := \frac{Vm}{\frac{Rr}{slip} + \iota \cdot Xr} \qquad Iout = 307.41064826 - 35.8132995\iota \qquad |Iout| = 309.48973986$$

$Pr := (|Iout|)^2 \cdot Rr \cdot 3$ $\qquad Pr = 4794.19506544$

Equivalent resistance representing the mechanical load:

$$Req := Rr \cdot \frac{(1 - slip)}{slip} \qquad Req = 0.8175193$$

Power dissipated by the resistor representing the energy converted to mechanical work. It should match the alternate calculation shown below:

$Pconv := (|Iout|)^2 \cdot Req \cdot 3$ $\qquad Pconv = 234915.55820643$

Input power (3 phase):

$Pin3 := Pin \cdot 3$ $\qquad Pin3 = 248533.33333333$

Stator power loss:

$Ps := (|Iin|)^2 \cdot Rs \cdot 3$ $\qquad Ps = 6586.78006146$ $\qquad \frac{Ps}{Pin3} = 2.65026022 \circ\%$

Core loss:

$Pm := \frac{(|Vm|)^2}{Rm} \cdot 3$ $\qquad Pm = 2236.8$ $\qquad \frac{Pm}{Pin3} = 0.9 \circ\%$

Power transferred across the air gap:

$Pag := Pin3 - Ps - Pm$ $\qquad Pag = 239709.75327187$ $\qquad \frac{Pag}{Pin3} = 96.44973978 \circ\%$

Rotor power loss:

$Prcu := Pag \cdot slip$ $\qquad Prcu = 4794.19506544$ $\qquad \frac{Prcu}{Pin3} = 1.9289948 \circ\%$

Power converted to mechanical energy:

$Pconv := Pag - Prcu$ $\qquad Pconv = 234915.55820643$ $\qquad \frac{Pconv}{Pin3} = 94.52074499 \circ\%$

Power loss due to friction and windage:

$Pfw := Pconv - Pout3$ $\qquad Pfw = 11235.55820643$ $\qquad \frac{Pfw}{Pin3} = 4.52074499 \circ\%$

Power at the shaft:

$HP = 300$ $\qquad Pout3 = 223680$ $\qquad \frac{Pout3}{Pin3} = 90 \circ\%$

The preceeding calculations were for full load. If we specified that our output mechanical load was less than full load, then all of the key model parameters will be the same except for the effective rotor resistance and hence the slip. We will now re-calculate the parameters for partial load. The approach entails calculating the Thevenin equivalent source as seen by the mechanical load equivalent R (Rr*(1-s)/s). We then use the same approach as used previously to calculate the load equivalent.

The Thevenin impedance is the parallel combination of the magnitizing impedance and the stator impedance plus the rotor impedance.

$$Zmag := \frac{1}{\frac{1}{Rm} + \frac{1}{\iota \cdot Xm}}$$

$$Zthev := \frac{1}{\frac{1}{Zmag} + \frac{1}{Rs + \iota \cdot Xs}} + Rr + \iota \cdot Xr \qquad Zthev = 0.02927788 + 0.10812394\iota$$

The Thevenin voltage is the voltage present across the magnetizing branch when the rotor circuit is open. This is a voltage divider formed by the stator impedance and the magnetizing impedance. The voltage divider formula is used to calculate the equivalent.

$$Vthev := V \cdot \frac{Zmag}{Zmag + Rs + \iota \cdot Xs} \qquad Vthev = 264.18674572 + 2.91184962\iota$$

Now, we have to write an equation in terms of the known quantities and the unknown mechanical load equivalent resistance. Our circuit now consists of the Thevenin voltage in series with the Thevenin equivalent impedance which then is in series with the mechanical load equivalent resistance Req to complete the circuit. We are given the output power, so we know that the equivalent resistance is related to this resistance by the square of the current through it. We use this to write our equation.

$$Pconv = (|Iout|)^2 \cdot Req$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{(|Zthev + Req|)^2}$$

$$(|Zthev + Req|)^2 = (Zrthev + Req)^2 + Zithev^2$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{(Zrthev + Req)^2 + Zithev^2}$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{Zrthev^2 + 2 \cdot Zrthev \cdot Req + Req^2 + Zithev^2}$$

$$(|Zthev|)^2 = Zrthev^2 + Zithev^2$$

$$(|Iout|)^2 = \frac{(|Vthev|)^2}{(|Zthev|)^2 + 2 \cdot Zrthev \cdot Req + Req^2}$$

$$Pconv = \frac{(|Vthev|)^2 \cdot Req}{(|Zthev|)^2 + 2 \cdot Zrthev \cdot Req + Req^2}$$

$$Pconv \cdot (|Zthev|)^2 + 2 \cdot Pconv \cdot Zthev \cdot Req + Pconv \cdot Req^2 = (|Vthev|)^2 \cdot Req$$

$$(|Zthev|)^2 + 2 \cdot Zthev \cdot Req + Req^2 = \frac{(|Vthev|)^2 \cdot Req}{Pconv}$$

$$0 = Req^2 + \left[2 \cdot Zrthev - \frac{(|Vthev|)^2}{Pout}\right] \cdot Req + (|Zthev|)^2$$

Set the mechanical output power based on the requested per-unit mechanical load. Assume that the friction and windage loss component remains constant.

$Load = 0.5$

$HP_{new} := HP \cdot Load$ $\quad HP_{new} = 150$

$Pout3_{new} := HP_{new} \cdot 745.6$ $\quad Pout3_{new} = 111840$

$Pconv_{new} := Pout3_{new} + Pfw \qquad Pconv_{new} = 123075.55820643$

Solve for the positive root which is the load equivalent resistance.

$$r1 := R1\left[1, 2 \cdot Re(Zthev) - \frac{(|Vthev|)^2}{\frac{Pconv_{new}}{3}}, (|Zthev|)^2\right]$$

$Req := r1 \qquad Req = 1.63524057$

Calculate the new slip for this operating point:

$$slip_{new} := \frac{Rr}{Rr + Req} \qquad slip_{new} = 0.01009978$$

$$Pout3_{new} := Pconv_{new} - Pfw \qquad Pout3_{new} = 111840$$

$$HP_{new} := \frac{Pout3_{new}}{745.6} \qquad HP_{new} = 150$$

Re-calculate the impedance at fundamental frequency of the motor:

$$Zm := Rs + \iota \cdot Xs + \frac{1}{\frac{1}{Rm} + \frac{1}{\iota \cdot Xm} + \frac{1}{\frac{Rr}{slip_{new}} + \iota \cdot Xr}} \qquad Zm = 0.53064322 + 0.81168309\iota$$

$$Ym := \frac{1}{Zm} \qquad Ym = 0.56426695 - 0.86311465\iota$$

$$Pin_{new} := V^2 \cdot Re(Ym) \qquad Pin_{new} = 43335.70184717$$

$$Qin_{new} := -V^2 \cdot Im(Ym) \qquad Qin_{new} = 66287.20531057$$

$$Pout_{new} := \frac{Pout3_{new}}{3} \qquad Pout_{new} = 37280$$

$$Sm_{new} := \sqrt{Pin_{new}{}^2 + Qin_{new}{}^2}$$

$$dDF_{new} := \frac{Pin_{new}}{Sm_{new}} \qquad dDF_{new} = 0.54719689$$

$Eff := \frac{Pout_{new}}{Pin_{new}}$

$Eff = 0.86026067$

$Speedm = 3528$ $Load = 0.5$

$Speedm_{new} := (1 - slip_{new}) \cdot Speed$

$Speedm_{new} = 3563.64080952$

$Speedm := Speedm_{new}$

$Speedm = 3563.64080952$

BRANCH NAME=ZSRC FROM=VSRC TO=NODE r = 1.0

$Zs := 1.0 + \iota \cdot 0.000001$ $|Zs| = 1$ $arg(Zs) \cdot \frac{180}{\pi} = 0.0000573$

$\omega s = 376.99111843$ $\omega r := \frac{Speedm \cdot \pi}{30}$ $\omega r = 373.18359291$ $slip := \frac{(\omega s - \omega r)}{\omega s}$ $slip = 0.01009978$

$\omega s(h) := \omega s \cdot h$ $h := 1$ $\omega s(1) = 376.99111843$

$Rmm := Rm$ $Lmm := \frac{Xm}{\omega s(1)}$ $Lmm = 0.00301071$ $Xmm(h) := 2 \cdot \pi \cdot f \cdot h \cdot Lmm$ $Xmm(1) = 1.13501234$

$Rr = 0.01668407$ $Lr := \frac{Xr}{\omega s(1)}$ $Lr = 0.00014664$ $Xr(h) := 2 \cdot \pi \cdot f \cdot h \cdot Lr$ $Xr(1) = 0.05528$

$Rs = 0.01382003$ $Ls := Lr$ $Ls = 0.00014664$ $Xs(h) := 2 \cdot \pi \cdot f \cdot h \cdot Ls$ $Xs(1) = 0.05528$

$Slip(h) := \frac{(\omega s(h) - \omega r)}{\omega s(h)}$ $Slip(1) = 0.01009978$

$Rrp(h) := Rr \cdot \frac{1 - Slip(h)}{Slip(h)}$ $Rrp(1) = 1.63524$ $Rrt(h) := Rrp(h) + Rr$

$Rrt(1) = 1.65192463$

$Yr(h) := \frac{1}{Rrp(h) + \iota \cdot Xr(h)}$ $Yr(1) = 0.61083275 - 0.0206495$ $Zr(h) := \frac{1}{Yr(h)}$

$Ym(h) := \frac{1}{Rmm + \iota \cdot 0} + \frac{1}{0 + i \cdot Xmm(h)}$ $Ym(1) = 0.01113696 - 0.881047$ $Zm(h) := \frac{1}{Ym(h)}$

$Zrm(h) := \frac{(Zr(h) \cdot Zm(h))}{Zr(h) + Zm(h)}$

$Zeq(h) := Rs + \iota \cdot Xs(h) + Zrm(h)$ $Zeq(1) = 0.53216953 + 0.80675446\iota$ $|Zeq(1)| = 0.96646633$

$$Zt(h) := \frac{Zs \cdot Zeq(h)}{Zs + Zeq(h)}$$

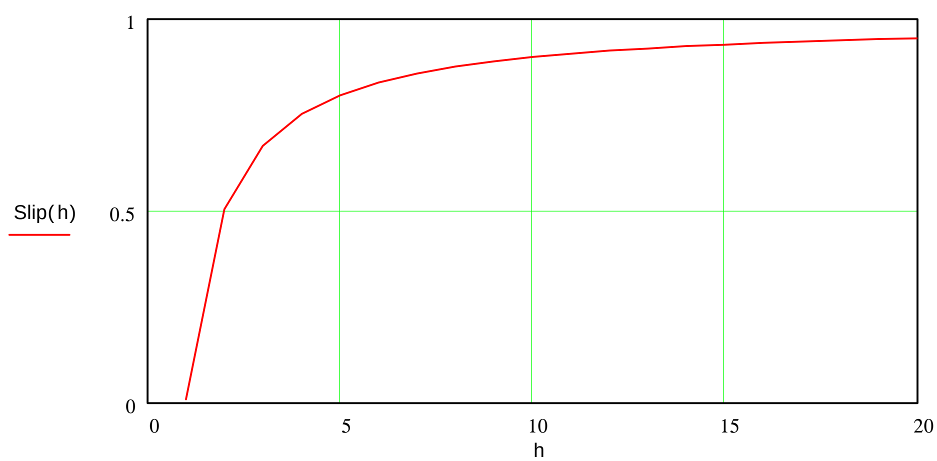
$Zt(1) = 0.489003 + 0.269062\iota$

$|Zt(1)| = 0.558139$ $\arg(Zt(1)) \cdot \frac{180}{\pi} = 28.82071473$

$h := 1, 2 .. 20$

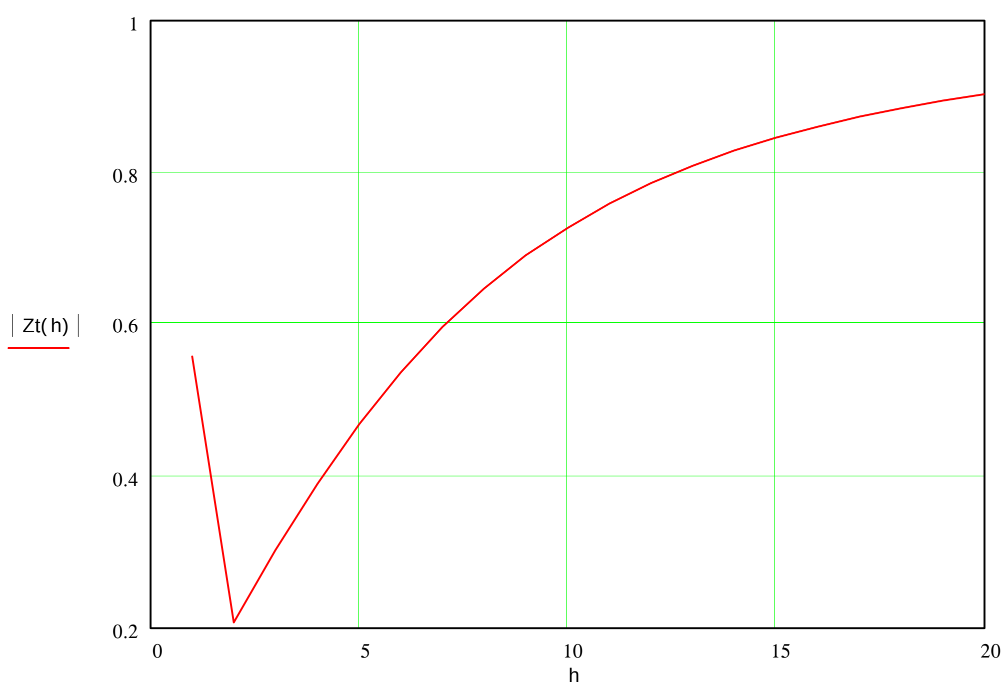
h	Slip(h)	$\frac{1 - Slip(h)}{Slip(h)}$	Rrp(h)
1	0.0101	98.0121	1.6352406
2	0.50505	0.98	0.0163504
3	0.67003	0.4925	0.0082163
4	0.75252	0.3289	0.0054867
5	0.80202	0.2469	0.0041185
6	0.83502	0.1976	0.0032965
7	0.85859	0.1647	0.002748
8	0.87626	0.1412	0.002356
9	0.89001	0.1236	0.0020618
10	0.90101	0.1099	0.001833
11	0.91001	0.0989	0.0016499
12	0.91751	0.0899	0.0015
13	0.92385	0.0824	0.0013751
14	0.92929	0.0761	0.0012694
15	0.93401	0.0707	0.0011788
16	0.93813	0.0659	0.0011003
17	0.94177	0.0618	0.0010316
18	0.94501	0.0582	0.0009709
19	0.9479	0.055	0.000917
20	0.9505	0.0521	0.0008688

```
! <Case_5e>
branch  name = cr2 from = noder  freqmult = 1
!
!           Freq(Hz) R(ohms) X(ohms)
!           ------------------------
 table= {
          {   60.0, 0.7470491, 0.0},
          {  120.0, 0.0146480, 0.0},
          {  180.0, 0.0073965, 0.0},
          {  240.0, 0.0049473, 0.0},
          {  300.0, 0.0037167, 0.0},
          {  360.0, 0.0029763, 0.0},
          {  420.0, 0.0024819, 0.0},
          {  480.0, 0.0021283, 0.0},
          {  540.0, 0.0018626, 0.0},
          {  600.0, 0.0016564, 0.0},
          {  660.0, 0.0014911, 0.0},
          {  720.0, 0.0013558, 0.0},
          {  780.0, 0.0012430, 0.0},
          {  840.0, 0.0011475, 0.0},
          {  900.0, 0.0010657, 0.0},
          {  960.0, 0.0009947, 0.0},
          { 1020.0, 0.0009326, 0.0},
          { 1080.0, 0.0008778, 0.0},
          { 1140.0, 0.0008291, 0.0},
          { 1200.0, 0.0007855, 0.0}
          }
```



$h := 1, 2 .. 20$

h	$\vert Zeq(h)\vert$	Zt(h)	$\vert Zt(h)\vert$	$arg(Zt(h))\cdot\frac{180}{\pi}$
1	0.966466	0.489 + 0.2691i	0.558139	28.82071473
2	0.217965	0.0691 + 0.1955i	0.207339	70.54521322
3	0.324686	0.1106 + 0.2821i	0.302961	68.59575
4	0.432384	0.1683 + 0.3525i	0.390572	64.47531207
5	0.540254	0.2335 + 0.4064i	0.468715	60.12140402
6	0.648182	0.3009 + 0.445i	0.537175	55.93722376
7	0.756134	0.3668 + 0.4703i	0.596397	52.04798887
8	0.8641	0.4289 + 0.4846i	0.647206	48.48941048
9	0.972074	0.4861 + 0.4905i	0.69059	45.2598527
10	1.080051	0.5378 + 0.49i	0.727563	42.34076555
11	1.188031	0.584 + 0.4849i	0.759075	39.70640875
12	1.296013	0.625 + 0.4766i	0.785978	37.32888772
13	1.403995	0.6612 + 0.4661i	0.809007	35.1807888
14	1.511978	0.6932 + 0.4543i	0.828788	33.23649003
15	1.61996	0.7214 + 0.4416i	0.845845	31.47272582
16	1.727942	0.7463 + 0.4286i	0.860614	29.86873609
17	1.835924	0.7683 + 0.4155i	0.873456	28.40619341
18	1.943904	0.7878 + 0.4026i	0.88467	27.06902089
19	2.051884	0.805 + 0.3899i	0.894504	25.84316555
20	2.159862	0.8204 + 0.3776i	0.903164	24.71636298



<Case5r.inf>

```
      Induction motor component info for LOAD

                          Vm
     ---[ Rs ]--[ Xs ]-----[ Rr ]--[ Xr ]--+
      +                   |                |
      |               +--+--+             +++      +-   -+
      |               |     |             | |      |1 - S|
      Vt             +++   +++            | | Rr x|-----|
      |          Rm | |   | | Xm          +++      |  S  |
      |              +++   +++             |       +-   -+
      +               |     |              |
     ----------------+-----+---------------+

      Rs,Xs=    0.0138119, j   0.0552475
      Rr,Xr=    0.0166853, j   0.0552475
      Rm,Xm=      89.7985, j     1.13494

      Vt   =        0.480  kV L-L         Pout =      111.840  kW
      Iin  =      285.788  Amps           HPout=      150.000  HP
      kVA  =      237.600  kVA            Slip =        0.010
      PF   =        0.547                 RPM  =     3563.639  RPM
      Pin  =      130.009  kW             Ploss=    18169.319  Watts
      Qin  =      198.875  kVAr           Eff  =       86.025
   Pstator =     3384.256  Watts          Prot =     1255.808  Watts
     Pconv =      123.079  kW             Pcore=     2289.888  Watts
       Pag =      124.335  kW             Pfw  =    11239.367  Watts
```

Parameter Summary:

$Rs = 0.01382003$ $\quad$ $Xs(1) = 0.05528011$

$Rr = 0.01668407$ $\quad$ $Xr(1) = 0.05528011$

$Rm = 89.79112948$ $\quad$ $Xm = 1.13501234$

SuperHarm Benchmarking - ISOURCE Modeling Equations

Electrotek Concepts - 6/10/98, TEG

This document illustrates the equations that define the operation of the ISOURCE model.

ISOURCE Model Verification:

Case 6c:

```
vsource name = vsource bus  = src    mag  = 13800   ang  = 60

branch  name=zsrc  from = src    to = node x = 0.01

isource name = isource bus  = node  freqmult = 60
table={
      {   1,    5.0,     -5},
      {   3,    0.115,   79},
      {   5,    3.94,   143},
      {   7,    3.045,  -54},
      {   9,    0.065,  -24},
      {  11,    1.425,   96},
      {  13,    0.68,  -107},
      {  17,    0.29,  -106},
      {  19,    0.355, -313}
      }
```

$Va := 13800$ $\quad Xs := 0.01$ $\quad Ls := \frac{Xs}{(2\cdot\pi\cdot 60)}$ $\quad Ls = 2.652582\cdot 10^{-5}$

$h := 1, 3 .. 19$ $\quad X(h) := 2\cdot\pi\cdot 60\cdot h\cdot Ls$

$V3mag := X(3)\cdot 0.115$ $\quad V3mag = 0.00345$ $\quad V3ang := 90 + 79$ $\quad V3ang = 169$

$V5mag := X(5)\cdot 3.94$ $\quad V5mag = 0.197$ $\quad V5ang := 90 + 143 - 360$ $\quad V5ang = -127$

$V7mag := X(7)\cdot 3.045$ $\quad V7mag = 0.21315$ $\quad V7ang := 90 - 54$ $\quad V7ang = 36$

$V9mag := X(9)\cdot 0.065$ $\quad V9mag = 0.00585$ $\quad V9ang := 90 - 24$ $\quad V9ang = 66$

$V11mag := X(11)\cdot 1.425$ $\quad V11mag = 0.15675$ $\quad V11ang := 90 + 96 - 360$ $\quad V11ang = -174$

$V13mag := X(13)\cdot 0.680$ $\quad V13mag = 0.0884$ $\quad V13ang := 90 - 107$ $\quad V13ang = -17$

$V17mag := X(17)\cdot 0.29$ $\quad V17mag = 0.0493$ $\quad V17ang := 90 - 106$ $\quad V17ang = -16$

$V19mag := X(19)\cdot 0.355$ $\quad V19mag = 0.06745$ $\quad V19ang := 90 - 313 + 360$ $\quad V19ang = 137$

SuperHarm Output Voltage

```
3          0.00345         169.0
5          0.197          -127.0
7          0.21315          36.0
9          0.00585          66.0
11         0.15675        -174.0
13         0.0884          -17.0
17         0.0493          -16.0
19         0.06745         137.0
```

SuperHarm Benchmarking - LINE Modeling Equations

Electrotek Concepts - 6/19/98, EWG/TEG

This document illustrates the equations that define the operation of the LINE model.

LINE Model Verification:

Case 7a:

The Calculation of Power Transmission Line Series Impedance and Shunt Capacitance Matrices from Line Geometry

This document demonstrates the calculation of line constant information based on the geometry of a transmission line. Both series impedance and shunt admittance matrices are calculated.

Note that the vectors of line data start with an unused element at the 0th position. This is done to facilitate the use of external user defined DLL's which require 0 based arrays but still accept 1 based indices.

Input data for the Case 7a:

```
line name=L1 nphase=3 length=100.0
  file = C:\ETKPROG\SuperHarm32\benchmark\Line\const_a.dat
  from = { bus1a, bus1b, bus1c }
    to = { bus2a, bus2b, bus2c }
xcoord = { -17.5, -13.5, -13.5 }
ycoord = {  96.0,  81.0, 113.0 }
 dcres = { 0.0863, 0.0863, 0.0863 }
   gmr = { 0.0404, 0.0404, 0.0404 }
  diam = { 1.212, 1.212, 1.212 }

Number of Phase  Conductors ...: 3
Number of Ground Conductors ...: 0
Phase  Conductors Transposed ..: FALSE
Ground Conductors Segmented  ..: FALSE
Frequency for Constants (Hz) ..: 60
Earth Restivity (ohm-meters) ..: 100
Units for Input Data ..........: Metric (SI)

ID     DC Res.        GMR    Diameter     X Coord     Y Coord  NB    Spacing
       ohms/km         cm          cm      meters      meters             cm
--  ----------  ----------  ----------  ----------  ----------  --  ----------
 P      0.0863      0.0404       1.212       -17.5          96   1           0
 P      0.0863      0.0404       1.212       -13.5          81   1           0
 P      0.0863      0.0404       1.212       -13.5         113   1           0
```

$$\text{dia} := \frac{1.212}{100} \qquad \text{rad} := \frac{\text{dia}}{2} \qquad \text{rad} = 6.06 \cdot 10^{-3}$$

$$hval := \begin{bmatrix} 0 & 0 \\ 96.0 & 96.0 \\ 81.0 & 81.0 \\ 113.0 & 113.0 \end{bmatrix} \quad xval := \begin{bmatrix} 0 & 0 \\ -17.5 & -17.5 \\ -13.5 & -13.5 \\ -13.5 & 13.5 \end{bmatrix} \quad rval := \begin{bmatrix} 0 & 0 \\ rad & rad \\ rad & rad \\ rad & rad \end{bmatrix}$$

$h := hval^{\langle 0 \rangle}$ $\quad x := xval^{\langle 0 \rangle}$ $\quad r := rval^{\langle 0 \rangle}$ $\quad r_p := r_1$

$f := 60$ $\quad GMR := \frac{0.0404}{100}$ $\quad \rho_e := 100$ $\quad R_{dc} := 8.6300 \cdot 10^{-5}$ $\quad \mu_0 := 4 \cdot \pi \cdot 10^{-7}$

$\omega := 2 \cdot \pi \cdot f$ $\quad \mu_r := 1$ $\quad \mu := \mu_0 \cdot \mu_r$ $\quad \varepsilon_0 := \frac{10^{-9}}{36 \cdot \pi}$ $\quad \varepsilon_0 = 8.84194 \cdot 10^{-12}$

$$h = \begin{bmatrix} 0 \\ 96 \\ 81 \\ 113 \end{bmatrix} \quad x = \begin{bmatrix} 0 \\ -17.5 \\ -13.5 \\ -13.5 \end{bmatrix} \quad r = \begin{bmatrix} 0 \\ 6.06 \cdot 10^{-3} \\ 6.06 \cdot 10^{-3} \\ 6.06 \cdot 10^{-3} \end{bmatrix} \quad r_1 = 6.06 \cdot 10^{-3} \quad GMR = 4.04 \cdot 10^{-4}$$

The formula for the internal inductance of a conductor neglecting skin effect is:

$$L_{internal.no.skin.effect} := \frac{\mu_0}{2 \cdot \pi} \cdot \ln\left(\frac{r_1}{GMR}\right) \qquad L_{internal.no.skin.effect} = 5.4161004 \cdot 10^{-7}$$

The internal resistance and inductance of a conductor can also be calculated by a function that takes skin effect into account. This involves modified Bessel functions which Mathcad cannot currently handle, so they utilize user defined DLL's (BERBEI.DLL, BESS.DLL, BESSP.DLL) in the \MATHCAD\USEREFI directory.

$A := \pi \cdot r_p^{\,2}$ $\quad L := 1$ $\quad \rho := \frac{R_{dc} \cdot A}{L}$ $\quad m := \sqrt{\frac{\omega \cdot \mu_0}{\rho}}$ $\quad mr := m \cdot r_p$

$A = 1.15371 \cdot 10^{-4}$ $\quad \rho = 9.95648 \cdot 10^{-9}$ $\quad m = 218.13107$ $\quad mr = 1.32187$

$ber(f) := Re(bess(f))$ $\qquad bei(f) := Im(bess(f))$

$berp(f) := Re(bessp(f))$ $\qquad beip(f) := Im(bessp(f))$

$ber(mr) = 0.95236$ $\qquad bei(mr) = 0.43452$

$berp(mr) = -0.14414$ $\qquad beip(mr) = 0.65492$

The following equation from Stevenson (Second Edition) is used to calculate the internal resistance and reactance of a cylindrical conductor:

$$Z_{internal} := \frac{\rho \cdot m}{2 \cdot \pi \cdot r_p} \cdot \left(\frac{ber(mr) + j \cdot bei(mr)}{beip(mr) - j \cdot berp(mr)}\right) \qquad Z_{internal} = 8.7055 \cdot 10^{-5} + 1.86837 \cdot 10^{-5} j$$

$$L_{internal} := \frac{Im(Z_{internal})}{\omega} \qquad L_{internal} = 4.95601 \cdot 10^{-8}$$

$$R_{internal} := Re(Z_{internal}) \qquad R_{internal} = 8.7055 \cdot 10^{-5}$$

Note that the internal inductance is only slightly lower than the uncorrected value. and the resistance is only slightly higher than the given DC value (6.2137E-5).

The external reactance calculations require information on the gemoetry of the conductors. The formula for calculating the distance between any two conductors i and k is:

$$d(i,k) := \sqrt{\left(x_i - x_k\right)^2 + \left(h_i - h_k\right)^2}$$

The formula for calculating the distance between any conductor i and image conductor k is:

$$D(i,k) := \sqrt{\left(x_i - x_k\right)^2 + \left(h_i + h_k\right)^2}$$

The external reactance due to the conductor itself (self impedance) for lossless earth is dependent on the geometry of the line and is given by the following expression:

$$ZSelf_{external}(i) := j \cdot \left(\omega \cdot \frac{\mu_0}{2 \cdot \pi} \cdot \ln\left(\frac{2 \cdot h_i}{r_i}\right)\right)$$

The external impedance due to neighboring conductors (mutual impedance) for lossless earth is also dependent on the geometry of the line and is given by the following expression:

$$ZMutual_{external}(i,k) := j \cdot \left(\omega \cdot \frac{\mu_0}{2 \cdot \pi} \cdot \ln\left(\frac{D(i,k)}{d(i,k)}\right)\right)$$

If the GMR is specified, then the internal and exernal self reactance calculations can be combined into a single expression. This is only useful if skin effect correction is not required for the internal inductance. This is usually not a problem since this value is very small compared to the external reactance and the change with frequency is also minor. Also, the skin effect calculation given above is only valid for cylindrical conductors. ACSR cables are better represented by a tubular geometry. In this case, the GMR from a table is more accurate.

$$Z_{Self}(i) := j \cdot \left(\omega \cdot \frac{\mu_0}{2 \cdot \pi} \cdot \ln\left(\frac{2 \cdot h_i}{GMR}\right)\right)$$

The earth contribution to the self and mutual reactances and resistances can be determined by using Carson's earth return correction formulas. These equations are quite lengthy and are almost impossible to implement in Mathcad. A custom DLL (CARSON.DLL in the \MATHCAD\USEREFI directory) has been developed to provide the function carsonZ. This function calculates the change in the impedance from the lossless case. The function requires the line geometry information as well as the frequency to evaluate the function at the given earth resistivity.

$$DeltaZ(i,k) := carsonZ\left(i,k,x,h,f,\rho_e\right)$$

The equations for calculating the self and mutual impedances of a line can now be defined:

$$Z_{ii}(i) := Z_{internal} + ZSelf_{external}(i) + DeltaZ(i,i)$$

or if GMR is specified:

$$Z_{ii}(i) := R_{internal} + Z_{Self}(i) + DeltaZ(i,i)$$

$$Z_{ik}(i,k) := ZMutual_{external}(i,k) + DeltaZ(i,k)$$

$$Z := \begin{bmatrix} Z_{ii}(1) & Z_{ik}(1,2) & Z_{ik}(1,3) \\ Z_{ik}(2,1) & Z_{ii}(2) & Z_{ik}(2,3) \\ Z_{ik}(3,1) & Z_{ik}(3,2) & Z_{ii}(3) \end{bmatrix}$$

$$Z = \begin{bmatrix} 1.35295\cdot10^{-4} + 1.11147\cdot10^{-3}j & 4.89235\cdot10^{-5} + 3.14541\cdot10^{-4}j & 4.7489\cdot10^{-5} + 3.07768\cdot10^{-4}j \\ 4.89235\cdot10^{-5} + 3.14541\cdot10^{-4}j & 1.36685\cdot10^{-4} + 1.10948\cdot10^{-3}j & 4.81506\cdot10^{-5} + 2.61127\cdot10^{-4}j \\ 4.7489\cdot10^{-5} + 3.07768\cdot10^{-4}j & 4.81506\cdot10^{-5} + 2.61127\cdot10^{-4}j & 1.33819\cdot10^{-4} + 1.11369\cdot10^{-3}j \end{bmatrix}$$

The shunt capacitance matrix can be found from the following relationships:

$$K := \frac{1}{2\cdot\pi\cdot\varepsilon_0} \qquad K = 1.8\cdot10^{10} \qquad \varepsilon_0 = 8.84194\cdot10^{-12}$$

$$P_{ii}(i) := K\cdot\ln\left(\frac{2\cdot h_i}{r_i}\right) \qquad P_{ik}(i,k) := K\cdot\ln\left(\frac{D(i,k)}{d(i,k)}\right)$$

The constant in front of the natural log expression can also be represented as follows:

$c := 299792500$ m/s (speed of light)

$$K := c^2\cdot\frac{\mu_0}{2\cdot\pi} \quad \text{or} \quad K := c^2\cdot2\cdot10^{-7} \qquad K = 1.79751\cdot10^{10}$$

The calculation using the permitivity of free space is identical to the calculation using the speed of light when the speed of light is set to 300000 km/s. Since the speed of light is slightly slower than that, the calculation using the correct value for c is more accurate. So we now redefine our functions for P:

$$P_{ii}(i) := c^2\cdot2\cdot10^{-7}\cdot\ln\left(\frac{2\cdot h_i}{r_i}\right) \qquad P_{ik}(i,k) := c^2\cdot2\cdot10^{-7}\cdot\ln\left(\frac{D(i,k)}{d(i,k)}\right)$$

$r_1 = 6.06\cdot10^{-3}$ $h_1 = 96$

$r_2 = 6.06\cdot10^{-3}$ $h_2 = 81$

$$\frac{(c^2\cdot2\cdot10^{-7})}{1000} = 1.79751\cdot10^{7}$$

$r_3 = 6.06\cdot10^{-3}$ $h_3 = 113$

$$P := \begin{bmatrix} P_{ii}(1) & P_{ik}(1,2) & P_{ik}(1,3) \\ P_{ik}(2,1) & P_{ii}(2) & P_{ik}(2,3) \\ P_{ik}(3,1) & P_{ik}(3,2) & P_{ii}(3) \end{bmatrix}$$

$$P = \begin{bmatrix} 1.86286\cdot 10^{11} & 4.37515\cdot 10^{10} & 4.46207\cdot 10^{10} \\ 4.37515\cdot 10^{10} & 1.83232\cdot 10^{11} & 3.23933\cdot 10^{10} \\ 4.46207\cdot 10^{10} & 3.23933\cdot 10^{10} & 1.89216\cdot 10^{11} \end{bmatrix} \qquad C := P^{-1}$$

$$C = \begin{bmatrix} 5.9374\cdot 10^{-12} & -1.20671\cdot 10^{-12} & -1.19356\cdot 10^{-12} \\ -1.20671\cdot 10^{-12} & 5.87315\cdot 10^{-12} & -7.20904\cdot 10^{-13} \\ -1.19356\cdot 10^{-12} & -7.20904\cdot 10^{-13} & 5.68983\cdot 10^{-12} \end{bmatrix} \qquad \omega C := j\cdot(\omega\cdot C)$$

$$\omega C = \begin{bmatrix} 2.23835\cdot 10^{-9}j & -4.54917\cdot 10^{-10}j & -4.49963\cdot 10^{-10}j \\ -4.54917\cdot 10^{-10}j & 2.21412\cdot 10^{-9}j & -2.71774\cdot 10^{-10}j \\ -4.49963\cdot 10^{-10}j & -2.71774\cdot 10^{-10}j & 2.14502\cdot 10^{-9}j \end{bmatrix} \qquad Cnfperkm := C\cdot 1000^{4}$$

SuperHarm Model Verification:

$$Cnfperkm = \begin{bmatrix} 5.9374 & -1.20671 & -1.19356 \\ -1.20671 & 5.87315 & -0.7209 \\ -1.19356 & -0.7209 & 5.68983 \end{bmatrix}$$

```
C nF/km:
 5.9374       -1.2067       -1.1936

-1.2067        5.8731       -0.72090

-1.1936       -0.72090       5.6898
```

$$Z = \begin{bmatrix} 1.35295\cdot10^{-4} + 1.11147\cdot10^{-3}j & 4.89235\cdot10^{-5} + 3.14541\cdot10^{-4}j & 4.7489\cdot10^{-5} + 3.07768\cdot10^{-4}j \\ 4.89235\cdot10^{-5} + 3.14541\cdot10^{-4}j & 1.36685\cdot10^{-4} + 1.10948\cdot10^{-3}j & 4.81506\cdot10^{-5} + 2.61127\cdot10^{-4}j \\ 4.7489\cdot10^{-5} + 3.07768\cdot10^{-4}j & 4.81506\cdot10^{-5} + 2.61127\cdot10^{-4}j & 1.33819\cdot10^{-4} + 1.11369\cdot10^{-3}j \end{bmatrix}$$

$$Zoperm := Z\cdot1000$$

SuperHarm Model Verification:

$$Zoperm = \begin{bmatrix} 0.1353 + 1.11147j & 0.04892 + 0.31454j & 0.04749 + 0.30777j \\ 0.04892 + 0.31454j & 0.13668 + 1.10948j & 0.04815 + 0.26113j \\ 0.04749 + 0.30777j & 0.04815 + 0.26113j & 0.13382 + 1.11369j \end{bmatrix}$$

```
Z Ohms/km - After  ground wire reduction:
 0.13530       0.048923      0.047489
 1.1115        0.31454       0.30777

 0.048923      0.13668       0.048151
 0.31454       1.1095        0.26113

 0.047489      0.048151      0.13382
 0.30777       0.26113       1.1137
```

SuperHarm Benchmarking - LINE Modeling Equations

Electrotek Concepts - 6/19/98, EWG/TEG

This document illustrates the equations that define the operation of the LINE model.

LINE Model Verification:

Case 7a:

This document demonstrates the phase to modal domain transformation of the matrices representing the series impedance and shunt admittance of a transmission line. The matrices themselves are obtained from a line constants program such as the AUX program in the EMTP or LineCalc, or the line constants calculations built into a program such as SuperHarm. Skin effect was neglected for this calculation and only the first few geometry independent terms of Carson's earth return equations were used.

Given the series admittance matrix Zp and the shunt admittance matrix Yp: Ohms/m

$$Z_p := \begin{bmatrix} 1.35295\cdot10^{-4}+1.11147\cdot10^{-3}\cdot i & 4.89235\cdot10^{-5}+3.14541\cdot10^{-4}\cdot i & 4.7489\cdot10^{-5}+3.07768\cdot10^{-4}\cdot i \\ 4.89235\cdot10^{-5}+3.14541\cdot10^{-4}\cdot i & 1.36685\cdot10^{-4}+1.10948\cdot10^{-3}\cdot i & 4.81506\cdot10^{-5}+2.61127\cdot10^{-4}\cdot i \\ 4.7489\cdot10^{-5}+3.07768\cdot10^{-4}\cdot i & 4.81506\cdot10^{-5}+2.61127\cdot10^{-4}\cdot i & 1.33819\cdot10^{-4}+1.11369\cdot10^{-3}\cdot i \end{bmatrix}$$

$$Y_p := \begin{bmatrix} 2.23835\cdot10^{-9}\cdot i & -4.54917\cdot10^{-10}\cdot i & -4.49963\cdot10^{-10}\cdot i \\ -4.54917\cdot10^{-10}\cdot i & 2.21412\cdot10^{-9}\cdot i & -2.71774\cdot10^{-10}\cdot i \\ -4.49963\cdot10^{-10}\cdot i & -2.71774\cdot10^{-10}\cdot i & 2.14502\cdot10^{-9}\cdot i \end{bmatrix}$$

Siemens/m

Calculate the product of Zp and Yp:

$$ZY := Z_p\cdot Y_p$$

$$ZY = \begin{bmatrix} -2.206\cdot10^{-12}+2.592\cdot10^{-13}i & -1.072\cdot10^{-13}+3.387\cdot10^{-14}i & -7.456\cdot10^{-14}+2.769\cdot10^{-14}i \\ -8.183\cdot10^{-14}+2.566\cdot10^{-14}i & -2.242\cdot10^{-12}+2.673\cdot10^{-13}i & -1.171\cdot10^{-13}+4.412\cdot10^{-14}i \\ -6.898\cdot10^{-14}+2.418\cdot10^{-14}i & -1.355\cdot10^{-13}+4.864\cdot10^{-14}i & -2.179\cdot10^{-12}+2.526\cdot10^{-13}i \end{bmatrix}$$

The next step is to find the transformation matrices which are used to convert the matrices from the phase domain into the modal domain. A complete discussion of the theory behind these operations can be found in the EMTP Theory Book by Herman W. Dommel, August 1986, BPA contract No. DE-AC79-81BP31364, section 4.1.5, pages 4-50 - 4-55.

The matrix Tv that diagonalizes the Zp*Yp matrix product is found by computing the eigenvectors of that matrix. Mathcad does not inherently support eigenvector/eigenvalue calculations on complex numbers so an extension DLL (CXEIGEN.DLL in the \MATHCAD\USEREFI directory) is used which contains the cxeigenvec and cxeigenval functions.

$$ZY_{ev} := \text{cxeigenvec}(ZY)$$

As a check to ensure that the eigenvectors were correctly computed, we retrieve the eigenvalues and determine that the eigenvalue/eigenvector equation (A - λ*I)x = 0 is satisfied.

$$ZY_{eval} := \text{cxeigenval}(ZY)$$

Define equations that evaluate A*x and λ*x:

$$fck1(i) := ZY \cdot \left(ZY_{ev}\right)^{\langle i \rangle} \qquad fck2(i) := ZY_{eval_{i,0}} \cdot \left(ZY_{ev}\right)^{\langle i \rangle}$$

For each eigenvector, check the identity of both equations:

$$fck1(0) = \begin{bmatrix} -1.298 \cdot 10^{-12} + 1.711 \cdot 10^{-13} i \\ -1.551 \cdot 10^{-12} + 1.99 \cdot 10^{-13} i \\ -1.31 \cdot 10^{-12} + 1.995 \cdot 10^{-13} i \end{bmatrix} \qquad fck2(0) = \begin{bmatrix} -1.298 \cdot 10^{-12} + 1.711 \cdot 10^{-13} i \\ -1.551 \cdot 10^{-12} + 1.99 \cdot 10^{-13} i \\ -1.31 \cdot 10^{-12} + 1.995 \cdot 10^{-13} i \end{bmatrix}$$

$$fck1(1) = \begin{bmatrix} 1.893 \cdot 10^{-12} - 2.156 \cdot 10^{-13} i \\ -7.668 \cdot 10^{-13} + 8.315 \cdot 10^{-14} i \\ -6.322 \cdot 10^{-13} + 5.552 \cdot 10^{-14} i \end{bmatrix} \qquad fck2(1) = \begin{bmatrix} 1.893 \cdot 10^{-12} - 2.156 \cdot 10^{-13} i \\ -7.668 \cdot 10^{-13} + 8.315 \cdot 10^{-14} i \\ -6.322 \cdot 10^{-13} + 5.552 \cdot 10^{-14} i \end{bmatrix}$$

$$fck1(2) = \begin{bmatrix} -6.86 \cdot 10^{-14} - 1.941 \cdot 10^{-14} i \\ 1.252 \cdot 10^{-12} - 1.489 \cdot 10^{-13} i \\ -1.662 \cdot 10^{-12} + 1.556 \cdot 10^{-13} i \end{bmatrix} \qquad fck2(2) = \begin{bmatrix} -6.86 \cdot 10^{-14} - 1.941 \cdot 10^{-14} i \\ 1.252 \cdot 10^{-12} - 1.489 \cdot 10^{-13} i \\ -1.662 \cdot 10^{-12} + 1.556 \cdot 10^{-13} i \end{bmatrix}$$

The resultant eigenvector system is:

$$ZY_{ev} = \begin{bmatrix} 0.538 + 2.439 \cdot 10^{-3} i & -0.886 + 2.609 \cdot 10^{-3} i & 0.032 + 0.013i \\ 0.643 + 5.119 \cdot 10^{-3} i & 0.359 + 8.862 \cdot 10^{-4} i & -0.602 + 9.858 \cdot 10^{-3} i \\ 0.545 - 8.452 \cdot 10^{-3} i & 0.295 + 6.757 \cdot 10^{-3} i & 0.798 + 6.945 \cdot 10^{-3} i \end{bmatrix}$$

If the eigenvector routine used did not return a normailzed set of vectors, the vectors can be normalized with the following equations:

$$N(i) := \sqrt{\left(ZY_{ev_{0,i}}\right)^2 + \left(ZY_{ev_{1,i}}\right)^2 + \left(ZY_{ev_{2,i}}\right)^2} \qquad N(0) = 1$$

$$N(1) = 1$$

$$N(2) = 1$$

$$ZY_n := \begin{bmatrix} \frac{ZY_{ev_{0,0}}}{N(0)} & \frac{ZY_{ev_{0,1}}}{N(1)} & \frac{ZY_{ev_{0,2}}}{N(2)} \\ \frac{ZY_{ev_{1,0}}}{N(0)} & \frac{ZY_{ev_{1,1}}}{N(1)} & \frac{ZY_{ev_{1,2}}}{N(2)} \\ \frac{ZY_{ev_{2,0}}}{N(0)} & \frac{ZY_{ev_{2,1}}}{N(1)} & \frac{ZY_{ev_{2,2}}}{N(2)} \end{bmatrix}$$

$$ZY_n = \begin{bmatrix} 0.538 + 2.439\cdot10^{-3}i & -0.886 + 2.609\cdot10^{-3}i & 0.032 + 0.013i \\ 0.643 + 5.119\cdot10^{-3}i & 0.359 + 8.862\cdot10^{-4}i & -0.602 + 9.858\cdot10^{-3}i \\ 0.545 - 8.452\cdot10^{-3}i & 0.295 + 6.757\cdot10^{-3}i & 0.798 + 6.945\cdot10^{-3}i \end{bmatrix}$$

The transformation matrix Tv is defined as equal to the normalized eigenvectors. This transformation matrix is used to transform the phase voltage vector into modal quantities.

$$T_v := ZY_n$$

The transformation matrix Ti is defined as the transpose of the inverse of Tv. Ti is used to transform the phase voltage vector into modal quantities. This matrix is output by the EMTP AUX LINE CONSTANTS routine and can be checked with this calculation.

$$T_i := \left(T_v^{-1}\right)^T$$

$$T_i = \begin{bmatrix} 0.467 + 4.486\cdot10^{-3}i & -0.846 + 1.74\cdot10^{-3}i & -5.802\cdot10^{-3} + 8.449\cdot10^{-3}i \\ 0.72 + 8.194\cdot10^{-3}i & 0.415 - 8.108\cdot10^{-4}i & -0.645 + 4.435\cdot10^{-3}i \\ 0.525 - 0.015i & 0.347 + 4.506\cdot10^{-3}i & 0.767 + 4.4\cdot10^{-3}i \end{bmatrix}$$

The following relationships are defined to convert the impedance and admittance matrices from the phase domain to the modal domain:

$$Z_m := T_i^T \cdot Z_p \cdot T_i \qquad Y_m := T_v^T \cdot Y_p \cdot T_v$$

As a check, the resultant matrices should now be fully decoupled (by definition):

$$Z_m = \begin{bmatrix} 2.31\cdot10^{-4} + 1.683\cdot10^{-3}i & 0 & 0 \\ 0 & 8.884\cdot10^{-5} + 7.935\cdot10^{-4}i & 0 \\ 0 & 0 & 8.598\cdot10^{-5} + 8.58\cdot10^{-4}i \end{bmatrix}$$

$$Y_m = \begin{bmatrix} -1.102\cdot10^{-12} + 1.431\cdot10^{-9}i & 0 & 0 \\ 0 & -2.709\cdot10^{-12} + 2.694\cdot10^{-9}i & 0 \\ 0 & 0 & 5.365\cdot10^{-12} + 2.426\cdot10^{-9}i \end{bmatrix}$$

We can now calculate the characteristic impedance of each mode:

$$Z_{c0} := \sqrt{\frac{Z_{m_{0,0}}}{Y_{m_{0,0}}}} \qquad Z_{c0} = 1086.948 - 74.655i \text{ Ohms} \qquad \left|Z_{c0}\right| = 1089.509 \qquad \arg\left(Z_{c0}\right)\cdot\frac{180}{\pi} = -3.929$$

$$Z_{c1} := \sqrt{\frac{Z_{m_{1,1}}}{Y_{m_{1,1}}}} \qquad Z_{c1} = 543.579 - 30.607i \text{ Ohms} \qquad \left|Z_{c1}\right| = 544.44 \qquad \arg\left(Z_{c1}\right)\cdot\frac{180}{\pi} = -3.223$$

$$Z_{c2} := \sqrt{\frac{Z_{m_{2,2}}}{Y_{m_{2,2}}}} \qquad Z_{c2} = 595.444 - 29.1i \text{ Ohms} \qquad \left|Z_{c2}\right| = 596.154 \qquad \arg\left(Z_{c2}\right)\cdot\frac{180}{\pi} = -2.798$$

The propagation constant γ (gamma) can also be calculated. The real part is the attenuation constant in units of Nepers/m. The imaginary part represents the velocity and is in units of radians per unit length.

$$G_0 := \sqrt{Z_{m_{0,0}}\cdot Y_{m_{0,0}}} \qquad G_0 = 1.057\cdot 10^{-7} + 1.556\cdot 10^{-6}i \qquad \left|G_0\right| = 1.559\cdot 10^{-6}$$

$$G_1 := \sqrt{Z_{m_{1,1}}\cdot Y_{m_{1,1}}} \qquad G_1 = 8.098\cdot 10^{-8} + 1.464\cdot 10^{-6}i \qquad \left|G_1\right| = 1.467\cdot 10^{-6}$$

$$G_2 := \sqrt{Z_{m_{2,2}}\cdot Y_{m_{2,2}}} \qquad G_2 = 7.38\cdot 10^{-8} + 1.445\cdot 10^{-6}i \qquad \left|G_2\right| = 1.446\cdot 10^{-6}$$

γ can also be found by taking the square root of the eigenvalues:

$$\overrightarrow{\sqrt{ZY_{eval}}} = \begin{bmatrix} 1.057\cdot 10^{-7} + 1.556\cdot 10^{-6}i \\ 8.098\cdot 10^{-8} + 1.464\cdot 10^{-6}i \\ 7.38\cdot 10^{-8} + 1.445\cdot 10^{-6}i \end{bmatrix}$$

Calculate the velocity of each mode:

$$V_0 := \frac{2\cdot\pi\cdot 60}{\mathrm{Im}\left(G_0\right)} \qquad V_0 = 2.423\cdot 10^{8} \text{ m/s}$$

$$V_1 := \frac{2\cdot\pi\cdot 60}{\mathrm{Im}\left(G_1\right)} \qquad V_1 = 2.574\cdot 10^{8} \text{ m/s}$$

$$V_2 := \frac{2\cdot\pi\cdot 60}{\mathrm{Im}\left(G_2\right)} \qquad V_2 = 2.61\cdot 10^{8} \text{ m/s}$$

```
DONE READING DISK FILE INTO EMTP CACHE.   NUMCRD =   64  CARDS.
SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.02
FOR INFORMATION, CONSULT THE EMTP RULE BOOK.          PROGRAM VERSION="V2.0"
    LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
--------------------------------------------------+----------------------------------------------------------------
------------------
DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
CHARACTER BY CHARACTER.
                                                  0         1         2         3         4         5         6
                                                  7         8
                                                  0         0         0         0         0         0         0
                                                  0         0
--------------------------------------------------+----------------------------------------------------------------
------------------
COMMENT CARD.                                     1C SUPERHARM LINE MODEL TEST - CP-LINE EMTP SETUP - CASE 7A
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C LINE NAME=L1 NPHASE=3 LENGTH=100.0
COMMENT CARD.                                     1C   FILE = C:\ETKPROG\SUPERHARM32\BENCHMARK\LINE\CONST_A.DAT
COMMENT CARD.                                     1C   FROM = { BUS1A, BUS1B, BUS1C }
COMMENT CARD.                                     1C     TO = { BUS2A, BUS2B, BUS2C }
COMMENT CARD.                                     1C XCOORD = { -17.5, -13.5, -13.5 }
COMMENT CARD.                                     1C YCOORD = {  96.0,  81.0, 113.0 }
COMMENT CARD.                                     1C  DCRES = { 0.0863, 0.0863, 0.0863 }
COMMENT CARD.                                     1C    GMR = { 0.0404, 0.0404, 0.0404 }
COMMENT CARD.                                     1C   DIAM = { 1.212, 1.212, 1.212 }
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C NUMBER OF PHASE  CONDUCTORS ...: 3
COMMENT CARD.                                     1C NUMBER OF GROUND CONDUCTORS ...: 0
COMMENT CARD.                                     1C PHASE  CONDUCTORS TRANSPOSED ..: FALSE
COMMENT CARD.                                     1C GROUND CONDUCTORS SEGMENTED  ..: FALSE
COMMENT CARD.                                     1C FREQUENCY FOR CONSTANTS (HZ) ..: 60
COMMENT CARD.                                     1C EARTH RESTIVITY (OHM-METERS) ..: 100
COMMENT CARD.                                     1C UNITS FOR INPUT DATA ..........: METRIC (SI)
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C ID     DC RES.         GMR    DIAMETER     X COORD     Y
COORD  NB     SPACING
COMMENT CARD.                                     1C        OHMS/KM          CM          CM      METERS
METERS              CM
COMMENT CARD.                                     1C --  ----------  ----------  ----------  ----------
----------  --  ----------
COMMENT CARD.                                     1C  P      0.0863      0.0404       1.212       -17.5
```

```
 96   1            0
 COMMENT CARD.                                     1C  P      0.0863      0.0404       1.212       -13.5
 81   1            0
 COMMENT CARD.                                     1C  P      0.0863      0.0404       1.212       -13.5
 113   1            0
 COMMENT CARD.                                     1C
 COMMENT CARD.                                     1C
                                                   1BEGIN NEW DATA CASE
+MARKER CARD PRECEDING NEW DATA CASE.
                                                   1LINE CONSTANTS
+REQUEST FOR LINE CONSTANTS
                                                   1LINE-PARAMETERS
 COMMENT CARD.                                     1C
 COMMENT CARD.                                     1C <-UNIT<-------------------------INOPT
                                                   1METRIC                               2
+METRIC   UNITS AND P.B.A.'S FORMAT ARE USED
 COMMENT CARD.                                     1C .....^..............................^
 COMMENT CARD.                                     1C               I
 COMMENT CARD.                                     1C               X       R
 COMMENT CARD.                                     1C P    S       R T    G  E       D       H       V       V
 S     A     N N
 COMMENT CARD.                                     1C H    K       E Y    M  A   CM  I   M   O    M  T    M  M
 CM  E DEG L     A B
 COMMENT CARD.                                     1C A    I       S P    R  C   IN  A  FT   R   FT  O   FT  I
 IN  P     P     M U
 COMMENT CARD.                                     1C
 S<---N<------I-E<------T-------M<------I<------W<------D<------A<----H<----E-N
 COMMENT CARD.                                     1C        1         2         3         4         5         6
 7         8
 COMMENT CARD.                                     1C
 345678901234567890123456789012345678901234567890123456789012345678901234567890
                                                   1  1       0.0863 2  0.0404   1.212   -17.5    96.0    96.0
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                   1  2       0.0863 2  0.0404   1.212   -13.5    81.0    81.0
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                   1  3       0.0863 2  0.0404   1.212   -13.5   113.0   113.0
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
 COMMENT CARD.                                     1C
                                                   1BLANK END OF CONDUCTOR DATA
```

```
+BLANK CARD TERMINATING LINE DATA
 COMMENT CARD.                                             1C
 COMMENT CARD.                                             1C ---RHO<-----FREQ<--------->ICPRNT-IZPRNT-C<----LEN
 <-PISM<-------> M F
                                                           1  100.00      60.0           111111 111111 1   100.0     00
                                                           1 1
+FREQUENCY CARD
 COMMENT CARD.                                             1C
 .....^.........^XXXXXXXXXXX^^^^^^X^^^^^^X^.......^X^^^^^^XXXXXXXXX.^.^
 COMMENT CARD.                                             1C
                                                           1.NODES              SEN_A     REC_A     SEN_B     REC_B
                                                           SEN_C     REC_C
+OPTIONAL NODE NAMES
 COMMENT CARD.                                             1C
 XXXXXXXXXXXXXXXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^
                                                           1BLANK CARD ENDIND FREQUENCY DATA
+BLANK CARD TERMINATING LINE PARAMETERS
```

```
////////////////////////////////////////////////////////////////////////////////

==========================      LINE-PARAMETERS    =============================

////////////////////////////////////////////////////////////////////////////////
```

METRIC S.I. UNITS ARE USED.
(CM ASSUMED FOR COND. DIAMETER AND BUNDLE SPACING)

RECORD OF SORTED INPUT DATA
(UNITS ARE THE SAME AS FOR INPUT)

PHASE	NUMBER	SKIN	DC RESIST	TYPE	PARAMETER	DIAMETER	X-COORD	Y-COORD	VREAL	VIMAGINARY
1	1	.0000	.08630	2	.04040	1.21200	-17.500	96.000	.000	.000
2	2	.0000	.08630	2	.04040	1.21200	-13.500	81.000	.000	.000
3	3	.0000	.08630	2	.04040	1.21200	-13.500	113.000	.000	.000

```
================================================================================

FOLLOWING MATRICES ARE FOR EARTH RESISTIVITY=  100.00 OHM-M AND FREQUENCY=        60.00 HZ. CORRECTION FACTOR=
.000001


---> P      INVERTED CAPACITANCE MATRIX (KM/F)
+                                                    FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                              ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

  1   1.86285E+08

  2   4.37514E+07   1.83231E+08

  3   4.46206E+07   3.23933E+07   1.89216E+08


---> C               CAPACITANCE MATRIX (F/KM)
+                                                    FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                              ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

  1   5.93741E-09

  2  -1.20671E-09   5.87316E-09

  3  -1.19357E-09  -7.20905E-10   5.68985E-09


---> P      INVERTED CAPACITANCE MATRIX (KM/F)
+     E                                              FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                              ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

  1   1.86285E+08

  2   4.37514E+07   1.83231E+08

  3   4.46206E+07   3.23933E+07   1.89216E+08
```

```
 ---> P      INVERTED CAPACITANCE MATRIX (KM/F)
+      S                                                      FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   2.66755E+08
       0.00000E+00

   1   3.95147E+06  -7.84129E+06
      -1.97852E+06   1.22578E+06

   2   3.95147E+06   1.45989E+08  -7.84129E+06
       1.97852E+06   4.26856E-10  -1.22578E+06



 ---> C               CAPACITANCE MATRIX (F/KM)
+      E                                                      FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   5.93741E-09

   2  -1.20671E-09   5.87316E-09

   3  -1.19357E-09  -7.20905E-10   5.68985E-09



 ---> C               CAPACITANCE MATRIX (F/KM)
+      S                                                      FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   3.75269E-09
       0.00000E+00

   1  -1.07775E-10   3.71457E-10
```

```
       4.91246E-11  -6.05059E-11

   2  -1.07775E-10   6.87386E-09   3.71457E-10
      -4.91246E-11   1.40018E-26   6.05059E-11



 ---> Z                   IMPEDANCE MATRIX (OHM/KM)
+                                                    FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.34513E-01
       1.11150E+00

   2   4.88980E-02   1.35907E-01
       3.14566E-01   1.10950E+00

   3   4.74596E-02   4.81230E-02   1.33032E-01
       3.07797E-01   2.61155E-01   1.11372E+00



 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+                                                    FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.14889E-01
      -1.01747E+00

   2  -2.16917E-02   1.12165E-01
       2.36499E-01  -9.96219E-01

   3  -2.06688E-02  -7.79223E-03   1.08604E-01
       2.27232E-01   1.71233E-01  -9.89092E-01



 ---> Z                   IMPEDANCE MATRIX (OHM/KM)
+      E                                             FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.34513E-01
```

```
     1.11150E+00

 2   4.88980E-02   1.35907E-01
     3.14566E-01   1.10950E+00

 3   4.74596E-02   4.81230E-02   1.33032E-01
     3.07797E-01   2.61155E-01   1.11372E+00
```

```
--------------------------------------------------------------------------------
                                 MODAL PARAMETERS AT FREQ =   6.00000E+01 HZ
--------------------------------------------------------------------------------
```

```
                              #####  MODAL WAVE QUANTITIES FROM Q COMPLEX  #####
                                         (Q ROTATED FOR GMODE = 0)

 MODE   RESISTANCE   REACTANCE  SUSCEPTANCE        SURGE IMPEDANCE(OHM)          VELOCITY   ATTENUATION
           OHM/KM      OHM/KM      S/KM         REAL        IMAG      LOSSLESS     KM/SEC     NEPER/KM
   1    2.26129E-01 1.66343E+00 1.44842E-06 1.07412E+03-7.26742E+01 1.07165E+03 2.42317E+05 1.05263E-04
   2    8.66331E-02 7.87732E-01 2.71381E-06 5.39577E+02-2.95815E+01 5.38765E+02 2.57453E+05 8.02787E-05
   3    8.67382E-02 8.54060E-01 2.43683E-06 5.92775E+02-3.00238E+01 5.92014E+02 2.60986E+05 7.31629E-05
```

```
                     EIGENVECTOR MATRIX Q FOR CURRENT TRANSFORMATION I(PHASE)=Q*I(MODE)
                                         (Q ROTATED FOR GMODE = 0)

                                        REAL COMPONENTS, ROW BY ROW

 4.63934E-01-8.42597E-01-5.90902E-03
 7.15471E-01 4.13183E-01-6.43656E-01
 5.22074E-01 3.45380E-01 7.65219E-01

                                      IMAGINARY COMPONENTS, ROW BY ROW

 4.19539E-03 1.17909E-03 8.54374E-03
 8.77845E-03-8.33922E-04 5.20382E-03
-1.45036E-02 4.55314E-03 3.59291E-03

                                    Z-SURGE LOSSLESS IN PHASE COMPONENTS
```

(FROM Z-SURGE MODAL LOSSLESS AND WITH Q REAL)

```
 7.40620E+02
 1.91540E+02 7.33651E+02
 1.91266E+02 1.51529E+02 7.47289E+02
```

```
 ---> Z                  IMPEDANCE MATRIX (OHM/KM)
+      S                                             FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                         ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                         ETC.

   0   2.30804E-01
       1.70058E+00

   1  -7.03690E-04  -5.14815E-03
       1.78852E-02  -3.33859E-02

   2   7.69796E-04   8.63238E-02   5.10261E-03
       1.53954E-02   8.17067E-01  -3.33875E-02
```

SYMMETRICAL COMPONENTS WAVE QUANTITIES

SEQUENCE	SURGE IMPEDANCE MAGNITUDE(OHM)	ANGLE(DEGR.)	ATTENUATION DB/KM	VELOCITY KM/S	WAVELENGTH KM	RESISTANCE OHM/KM	REACTANCE OHM/KM	SUSCEPTANCE S/KM
ZERO	1.10140E+03	-3.86449E+00	9.12163E-04	2.42495E+05	4.04158E+03	2.30804E-01	1.70058E+00	1.41473E-06
POSITIVE	5.63077E+02	-3.01549E+00	6.66728E-04	2.58722E+05	4.31203E+03	8.63238E-02	8.17067E-01	2.59139E-06

```
 ---> Y         INVERTED IMPEDANCE MATRIX (S-KM)
+      E                                             FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                              ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.14889E-01
      -1.01747E+00
```

```
  2  -2.16917E-02   1.12165E-01
      2.36499E-01  -9.96219E-01

  3  -2.06688E-02  -7.79223E-03   1.08604E-01
      2.27232E-01   1.71233E-01  -9.89092E-01


 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+      S                                                FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

  0   7.84507E-02
     -5.77619E-01

  1  -3.57854E-03   3.01942E-03
      1.26708E-02  -5.03125E-02

  2  -2.34321E-03   1.28603E-01   1.78348E-02
      1.12054E-02  -1.21258E+00  -4.70753E-02

 ================================================================================


                     :::::  PI-NOMINAL (APPROXIMATION) USING LUMPED LINE PARAMETERS  :::::


 MATRICES FOR LINE LENGTH = 1.000000E+02   KM
 THIS IS NOT AN EXACT EQUIVALENT PI-CIRCUIT, BUT AN APPROXIMATE PI-CIRCUIT


 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

                      PI-NOMINAL CIRCUIT FOR THE TOTAL LINE LENGTH WILL BE PUNCHED IN LU7


 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
```

```
                                                     1BLANK CARD ENDING LINE CONSTANTS CASE
+BLANK CARD TERMINATING 'LINE CONSTANTS' CASES.

 CORE STORAGE FIGURES FOR PRECEDING DATA CASE NOW COMPLETED.  --------------------------------------  PRESENT
 PROGRAM
 A VALUE OF  -9999 INDICATES DEFAULT WITH NO FIGURE AVAILABLE.                                        FIGURE
 LIMIT (NAME)
     SIZE LIST  1.   TOTAL STORAGE SPACE ALLOCATED FOR THE EMTP AUXILIARY PROGRAMS
     281040(LTLABL)
 TIMING FIGURES (DECIMAL) CHARACTERIZING CASE SOLUTION SPEED.  -------------------------------------    CP SEC
 I/O SEC   SUM SEC
                                                                                            TOTALS       .110
                                                                                            .000      .110


 SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
 DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.02
 FOR INFORMATION, CONSULT THE EMTP RULE BOOK.           PROGRAM VERSION="V2.0"
     LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
 --------------------------------------------------+------------------------------------------------------------
 ------------------
 DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
 CHARACTER BY CHARACTER.
                                                   0         1         2         3         4         5         6
                                                   7         8
                                                   0         0         0         0         0         0         0
                                                   0         0
 --------------------------------------------------+------------------------------------------------------------
 ------------------
                                                   1BLANK END OF EMTP DATA
+BLANK TERMINATION-OF-RUN CARD.
```

```
DONE READING DISK FILE INTO EMTP CACHE.   NUMCRD =   71  CARDS.
SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.08
FOR INFORMATION, CONSULT THE EMTP RULE BOOK.           PROGRAM VERSION="V2.0"
    LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
--------------------------------------------------+-------------------------------------------------------------
------------------
DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
CHARACTER BY CHARACTER.
                                                  0         1         2         3         4         5         6
                                                  7         8
                                                  0         0         0         0         0         0         0
                                                  0         0
--------------------------------------------------+-------------------------------------------------------------
------------------
COMMENT CARD.                                     1C SUPERHARM LINE MODEL TEST - CP-LINE EMTP SETUP - CASE 7B
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C LINE NAME=L1 NPHASE=3 LENGTH=100.0  FREQ = 60
COMMENT CARD.                                     1C   TRANSPOSED = NO  SEGMENTED = NO  METRIC = YES
COMMENT CARD.                                     1C   FILE = C:\ETKPROG\SUPERHARM32\BENCHMARK\LINE\CONST_B.DAT
COMMENT CARD.                                     1C   FROM = { BUS1A, BUS1B, BUS1C, G1 }
COMMENT CARD.                                     1C     TO = { BUS2A, BUS2B, BUS2C, G2 }
COMMENT CARD.                                     1C XCOORD = { -17.5, -13.5, -13.5, -8.0,  8.0 }
COMMENT CARD.                                     1C YCOORD = {  96.0,  81.0, 113.0,131.0,131.0 }
COMMENT CARD.                                     1C  DCRES = { 0.0863, 0.0863, 0.0863, 3.4468, 3.4468}
COMMENT CARD.                                     1C    GMR = { 0.0404, 0.0404, 0.0404, 0.0145, 0.0145}
COMMENT CARD.                                     1C   DIAM = { 1.212, 1.212, 1.212, 0.349, 0.349 }
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C NUMBER OF PHASE  CONDUCTORS ...: 3
COMMENT CARD.                                     1C NUMBER OF GROUND CONDUCTORS ...: 2
COMMENT CARD.                                     1C PHASE  CONDUCTORS TRANSPOSED ..: FALSE
COMMENT CARD.                                     1C GROUND CONDUCTORS SEGMENTED  ..: FALSE
COMMENT CARD.                                     1C FREQUENCY FOR CONSTANTS (HZ) ..: 60
COMMENT CARD.                                     1C EARTH RESTIVITY (OHM-METERS) ..: 100
COMMENT CARD.                                     1C UNITS FOR INPUT DATA ..........: METRIC (SI)
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C ID     DC RES.         GMR    DIAMETER     X COORD     Y
COORD  NB     SPACING
COMMENT CARD.                                     1C        OHMS/KM          CM          CM      METERS
METERS              CM
COMMENT CARD.                                     1C --  ----------  ----------  ----------  ----------
----------  --  ----------
```

```
 COMMENT CARD.                                   1C  P      0.0863     0.0404      1.212      -17.5
 96   1          0
 COMMENT CARD.                                   1C  P      0.0863     0.0404      1.212      -13.5
 81   1          0
 COMMENT CARD.                                   1C  P      0.0863     0.0404      1.212      -13.5
 113   1          0
 COMMENT CARD.                                   1C  G      3.4468     0.0145      0.349         -8
 131   1          0
 COMMENT CARD.                                   1C  G      3.4468     0.0145      0.349          8
 131   1          0
 COMMENT CARD.                                   1C
 COMMENT CARD.                                   1C
                                                 1BEGIN NEW DATA CASE
+MARKER CARD PRECEDING NEW DATA CASE.
                                                 1LINE CONSTANTS
+REQUEST FOR LINE CONSTANTS
                                                 1LINE-PARAMETERS
 COMMENT CARD.                                   1C
 COMMENT CARD.                                   1C <-UNIT<-------------------------INOPT
                                                 1METRIC                               2
+METRIC   UNITS AND P.B.A.'S FORMAT ARE USED
 COMMENT CARD.                                   1C .....^..............................^
 COMMENT CARD.                                   1C               I
 COMMENT CARD.                                   1C               X       R
 COMMENT CARD.                                   1C P    S      R T    G  E      D      H      V      V
 S     A     N N
 COMMENT CARD.                                   1C H    K      E Y    M  A   CM  I   M   O    M  T    M  M
 CM  E DEG L     A B
 COMMENT CARD.                                   1C A    I      S P    R  C   IN  A  FT   R   FT  O   FT  I
 IN  P     P     M U
 COMMENT CARD.                                   1C
 S<---N<------I-E<------T-------M<------I<------W<------D<------A<----H<----E-N
 COMMENT CARD.                                   1C       1         2         3         4         5         6
 7         8
 COMMENT CARD.                                   1C
 345678901234567890123456789012345678901234567890123456789012345678901234567890
                                                 1  0       3.4468 2  0.0145   0.349    -8.0   131.0   131.0
 COMMENT CARD.                                   1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                 1  0       3.4468 2  0.0145   0.349     8.0   131.0   131.0
 COMMENT CARD.                                   1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
```

```
                                                  1  1      0.0863 2  0.0404   1.212   -17.5    96.0    96.0
 COMMENT CARD.                                    1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                  1  2      0.0863 2  0.0404   1.212   -13.5    81.0    81.0
 COMMENT CARD.                                    1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                  1  3      0.0863 2  0.0404   1.212   -13.5   113.0   113.0
 COMMENT CARD.                                    1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
 COMMENT CARD.                                    1C
                                                  1BLANK END OF CONDUCTOR DATA
+BLANK CARD TERMINATING LINE DATA
 COMMENT CARD.                                    1C
 COMMENT CARD.                                    1C ---RHO<-----FREQ<--------->ICPRNT-IZPRNT-C<----LEN
 <-PISM<-------> M F
                                                  1  100.00      60.0           111111 111111 1   100.0     00
                                                  1 1
+FREQUENCY CARD
 COMMENT CARD.                                    1C
 .....^.........^XXXXXXXXXXXX^^^^^^X^^^^^^^X^.......^X^^^^^^XXXXXXXXX.^.^
 COMMENT CARD.                                    1C
                                                  1.NODES              SEN_A     REC_A     SEN_B     REC_B
                                                  SEN_C     REC_C
+OPTIONAL NODE NAMES
 COMMENT CARD.                                    1C
 XXXXXXXXXXXXXXXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^
                                                  1BLANK CARD ENDIND FREQUENCY DATA
+BLANK CARD TERMINATING LINE PARAMETERS



 ////////////////////////////////////////////////////////////////////////////////

 ==========================      LINE-PARAMETERS    =============================

 ////////////////////////////////////////////////////////////////////////////////
```

METRIC S.I. UNITS ARE USED.
(CM ASSUMED FOR COND. DIAMETER AND BUNDLE SPACING)

RECORD OF SORTED INPUT DATA
(UNITS ARE THE SAME AS FOR INPUT)

PHASE	NUMBER	SKIN	DC RESIST	TYPE	PARAMETER	DIAMETER	X-COORD	Y-COORD	VREAL	VIMAGINARY
1	1	.0000	.08630	2	.04040	1.21200	-17.500	96.000	.000	.000
2	2	.0000	.08630	2	.04040	1.21200	-13.500	81.000	.000	.000
3	3	.0000	.08630	2	.04040	1.21200	-13.500	113.000	.000	.000
4	0	.0000	3.44680	2	.01450	.34900	-8.000	131.000	.000	.000
5	0	.0000	3.44680	2	.01450	.34900	8.000	131.000	.000	.000

==

FOLLOWING MATRICES ARE FOR EARTH RESISTIVITY= 100.00 OHM-M AND FREQUENCY=   60.00 HZ. CORRECTION FACTOR=
.000001

---> P   INVERTED CAPACITANCE MATRIX (KM/F)
+   FOR THE SYSTEM OF PHYSICAL CONDUCTORS
ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

1	1.86285E+08				
2	4.37514E+07	1.83231E+08			
3	4.46206E+07	3.23933E+07	1.89216E+08		
4	3.29831E+07	2.58641E+07	4.60597E+07	2.14251E+08	
5	2.98920E+07	2.45333E+07	3.89591E+07	5.02874E+07	2.14251E+08

---> C   CAPACITANCE MATRIX (F/KM)
+   FOR THE SYSTEM OF PHYSICAL CONDUCTORS
ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

```
  1   6.03123E-09

  2  -1.13631E-09   5.92614E-09

  3  -1.02738E-09  -5.96678E-10   5.98558E-09

  4  -4.73408E-10  -3.33995E-10  -9.00408E-10   5.19762E-09

  5  -4.13421E-10  -3.33157E-10  -6.65410E-10  -9.51922E-10   5.10767E-09


 ---> P      INVERTED CAPACITANCE MATRIX (KM/F)
+      E                                              FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

  1   1.78784E+08

  2   3.77497E+07   1.78425E+08

  3   3.44501E+07   2.42660E+07   1.75400E+08


 ---> P      INVERTED CAPACITANCE MATRIX (KM/F)
+      S                                              FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

  0   2.41847E+08
      0.00000E+00

  1   4.56842E+06  -7.26553E+06
      1.82577E+06   1.03176E+06

  2   4.56842E+06   1.45381E+08  -7.26553E+06
     -1.82577E+06  -4.65661E-10  -1.03176E+06


 ---> C               CAPACITANCE MATRIX (F/KM)
```

```
+      E                                            FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   6.03123E-09

   2  -1.13631E-09   5.92614E-09

   3  -1.02738E-09  -5.96678E-10   5.98558E-09



 ---> C               CAPACITANCE MATRIX (F/KM)
+      S                                            FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   4.14074E-09
       0.00000E+00

   1  -1.36599E-10   3.48569E-10
      -4.86029E-11  -4.57340E-11

   2  -1.36599E-10   6.90111E-09   3.48569E-10
       4.86029E-11   0.00000E+00   4.57340E-11



 ---> Z                   IMPEDANCE MATRIX (OHM/KM)
+                                                   FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.34513E-01
       1.11150E+00

   2   4.88980E-02   1.35907E-01
       3.14566E-01   1.10950E+00

   3   4.74596E-02   4.81230E-02   1.33032E-01
       3.07797E-01   2.61155E-01   1.11372E+00

   4   4.66868E-02   4.73288E-02   4.59862E-02   3.49207E+00
```

```
        2.53867E-01   2.28230E-01   3.04404E-01   1.19326E+00

    5   4.66642E-02   4.73104E-02   4.59698E-02   4.52577E-02   3.49207E+00
        2.40508E-01   2.22298E-01   2.74358E-01   3.17789E-01   1.19326E+00


 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+                                                      FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

    1   1.23775E-01
       -1.02203E+00

    2  -1.39214E-02   1.18960E-01
        2.32248E-01  -1.00017E+00

    3  -7.60440E-03   3.63323E-03   1.27799E-01
        2.21345E-01   1.65704E-01  -9.96589E-01

    4  -3.43992E-02  -2.91789E-02  -5.28224E-02   2.63095E-01
        1.47223E-02   1.36828E-02   1.90452E-02  -8.04741E-02

    5  -3.27737E-02  -2.97211E-02  -4.54750E-02  -9.83223E-03   2.62345E-01
        1.46945E-02   1.40000E-02   1.82675E-02  -1.17959E-02  -8.15018E-02


 ---> Z                   IMPEDANCE MATRIX (OHM/KM)
+      E                                               FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

    1   1.57998E-01
        1.08843E+00

    2   6.99406E-02   1.54749E-01
        2.93021E-01   1.08940E+00

    3   7.56105E-02   7.33637E-02   1.66750E-01
        2.81719E-01   2.36778E-01   1.08429E+00
```

```
--------------------------------------------------------------------------------
                                 MODAL PARAMETERS AT FREQ =   6.00000E+01 HZ
--------------------------------------------------------------------------------


                              #####  MODAL WAVE QUANTITIES FROM Q COMPLEX  #####
                                          (Q ROTATED FOR GMODE = 0)

 MODE   RESISTANCE   REACTANCE  SUSCEPTANCE       SURGE IMPEDANCE(OHM)          VELOCITY   ATTENUATION
           OHM/KM      OHM/KM      S/KM         REAL        IMAG      LOSSLESS     KM/SEC     NEPER/KM
   1    3.02685E-01 1.60468E+00 1.58470E-06 1.01071E+03-9.44903E+01 1.00628E+03 2.35373E+05 1.49739E-04
   2    8.65804E-02 7.86469E-01 2.71818E-06 5.38712E+02-2.95635E+01 5.37901E+02 2.57452E+05 8.03587E-05
   3    8.71463E-02 8.51751E-01 2.46412E-06 5.88697E+02-3.00376E+01 5.87930E+02 2.59883E+05 7.40163E-05


                       EIGENVECTOR MATRIX Q FOR CURRENT TRANSFORMATION I(PHASE)=Q*I(MODE)
                                          (Q ROTATED FOR GMODE = 0)

                                         REAL COMPONENTS, ROW BY ROW

 4.72695E-01 8.36776E-01 8.28641E-02
 5.73193E-01-4.39487E-01 6.94660E-01
 6.69068E-01-3.26418E-01-7.14415E-01

                                       IMAGINARY COMPONENTS, ROW BY ROW

-6.77731E-03 3.41949E-03-1.32275E-02
 1.54888E-02 9.49887E-03-1.59005E-03
-8.37905E-03-2.65195E-03-3.71000E-03


                                     Z-SURGE LOSSLESS IN PHASE COMPONENTS
                                  (FROM Z-SURGE MODAL LOSSLESS AND WITH Q REAL)

 7.17779E+02
 1.71944E+02 7.16481E+02
 1.62082E+02 1.26554E+02 7.09787E+02


--------------------------------------------------------------------------------
```

```
 ---> Z                  IMPEDANCE MATRIX (OHM/KM)
+      S                                            FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   3.05776E-01
       1.62838E+00

   1  -5.85086E-03  -5.57594E-03
       1.22907E-02  -3.30088E-02

   2   3.62419E-03   8.68609E-02   4.52558E-03
       2.24929E-02   8.16865E-01  -3.33904E-02


                                 #####  SYMMETRICAL COMPONENTS WAVE QUANTITIES  #####
 SEQUENCE      SURGE IMPEDANCE       ATTENUATION   VELOCITY    WAVELENGTH   RESISTANCE    REACTANCE SUSCEPTANCE
         MAGNITUDE(OHM) ANGLE(DEGR.)   DB/KM          KM/S          KM    OHM/KM       OHM/KM        S/KM
   ZERO    1.03024E+03 -5.31754E+00  1.29457E-03  2.35429E+05  3.92381E+03  3.05776E-01  1.62838E+00  1.56102E-06
 POSITIVE  5.61916E+02 -3.03485E+00  6.72276E-04  2.58238E+05  4.30396E+03  8.68609E-02  8.16865E-01  2.60166E-06


 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+      E                                            FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                              ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.23775E-01
      -1.02203E+00

   2  -1.39214E-02   1.18960E-01
       2.32248E-01  -1.00017E+00

   3  -7.60440E-03   3.63323E-03   1.27799E-01
       2.21345E-01   1.65704E-01  -9.96589E-01
```

```
 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+     S                                         FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   1.11583E-01
      -5.93398E-01

   1  -6.77934E-03   2.39964E-03
       8.10685E-03  -4.97060E-02

   2  -2.55427E-03   1.29476E-01   1.70590E-02
       1.68568E-02  -1.21270E+00  -4.75147E-02

 ================================================================================


                     :::::  PI-NOMINAL (APPROXIMATION) USING LUMPED LINE PARAMETERS  :::::


 MATRICES FOR LINE LENGTH = 1.000000E+02   KM
 THIS IS NOT AN EXACT EQUIVALENT PI-CIRCUIT, BUT AN APPROXIMATE PI-CIRCUIT


 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

                      PI-NOMINAL CIRCUIT FOR THE TOTAL LINE LENGTH WILL BE PUNCHED IN LU7


 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~



                                                  1BLANK CARD ENDING LINE CONSTANTS CASE
+BLANK CARD TERMINATING 'LINE CONSTANTS' CASES.

 CORE STORAGE FIGURES FOR PRECEDING DATA CASE NOW COMPLETED.  ---------------------------------------  PRESENT
 PROGRAM
 A VALUE OF  -9999 INDICATES DEFAULT WITH NO FIGURE AVAILABLE.                                         FIGURE
 LIMIT (NAME)
```

```
    SIZE LIST  1.   TOTAL STORAGE SPACE ALLOCATED FOR THE EMTP AUXILIARY PROGRAMS
    281040(LTLABL)
TIMING FIGURES (DECIMAL) CHARACTERIZING CASE SOLUTION SPEED.  -------------------------------------    CP SEC
I/O SEC   SUM SEC
                                                                                           TOTALS       .000
                                                                                           .000      .000


SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.09
FOR INFORMATION, CONSULT THE EMTP RULE BOOK.           PROGRAM VERSION="V2.0"
    LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
--------------------------------------------------+------------------------------------------------------------
------------------
DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
CHARACTER BY CHARACTER.
                                                  0         1         2         3         4         5         6
                                                  7         8
                                                  0         0         0         0         0         0         0
                                                  0         0
--------------------------------------------------+------------------------------------------------------------
------------------
                                                  1BLANK END OF EMTP DATA
+BLANK TERMINATION-OF-RUN CARD.
```

```
DONE READING DISK FILE INTO EMTP CACHE.   NUMCRD =   71  CARDS.
SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.12
FOR INFORMATION, CONSULT THE EMTP RULE BOOK.           PROGRAM VERSION="V2.0"
    LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
--------------------------------------------------+------------------------------------------------------------
------------------
DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
CHARACTER BY CHARACTER.
                                                  0         1         2         3         4         5         6
                                                  7         8
                                                  0         0         0         0         0         0         0
                                                  0         0
--------------------------------------------------+------------------------------------------------------------
------------------
COMMENT CARD.                                     1C SUPERHARM LINE MODEL TEST - CP-LINE EMTP SETUP - CASE 7C
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C LINE NAME=L1 NPHASE=3 EARTH=90 LENGTH=100.0  FREQ = 50
COMMENT CARD.                                     1C   TRANSPOSED = NO   SEGMENTED = YES  METRIC = NO
COMMENT CARD.                                     1C   FILE = C:\ETKPROG\SUPERHARM32\BENCHMARK\LINE\CONST_C.DAT
COMMENT CARD.                                     1C   FROM = { BUS1A, BUS1B, BUS1C, G1 }
COMMENT CARD.                                     1C     TO = { BUS2A, BUS2B, BUS2C, G2 }
COMMENT CARD.                                     1C XCOORD = { -17.5, -13.5, -13.5, -8.0,  8.0 }
COMMENT CARD.                                     1C YCOORD = {  96.0,  81.0, 113.0,131.0,131.0 }
COMMENT CARD.                                     1C  DCRES = { 0.0863, 0.0863, 0.0863, 3.4468, 3.4468 }
COMMENT CARD.                                     1C    GMR = { 0.0404, 0.0404, 0.0404, 0.0145, 0.0145 }
COMMENT CARD.                                     1C   DIAM = { 1.212, 1.212, 1.212, 0.349, 0.349 }
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C NUMBER OF PHASE  CONDUCTORS ...: 3
COMMENT CARD.                                     1C NUMBER OF GROUND CONDUCTORS ...: 2
COMMENT CARD.                                     1C PHASE  CONDUCTORS TRANSPOSED ..: TRUE
COMMENT CARD.                                     1C GROUND CONDUCTORS SEGMENTED  ..: TRUE
COMMENT CARD.                                     1C FREQUENCY FOR CONSTANTS (HZ) ..: 50
COMMENT CARD.                                     1C EARTH RESTIVITY (OHM-METERS) ..: 90
COMMENT CARD.                                     1C UNITS FOR INPUT DATA ..........: ENGLISH
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C ID     DC RES.         GMR    DIAMETER     X COORD     Y
COORD  NB     SPACING
COMMENT CARD.                                     1C        OHMS/MI        FEET      INCHES        FEET
FEET          INCHES
COMMENT CARD.                                     1C --  ----------  ----------  ----------  ----------
----------  --  ----------
```

```
 COMMENT CARD.                                          1C  P      0.0863      0.0404       1.212       -17.5
 96   1           0
 COMMENT CARD.                                          1C  P      0.0863      0.0404       1.212       -13.5
 81   1           0
 COMMENT CARD.                                          1C  P      0.0863      0.0404       1.212       -13.5
 113   1           0
 COMMENT CARD.                                          1C  G      3.4468      0.0145       0.349          -8
 131   1           0
 COMMENT CARD.                                          1C  G      3.4468      0.0145       0.349           8
 131   1           0
 COMMENT CARD.                                          1C
 COMMENT CARD.                                          1C
                                                        1BEGIN NEW DATA CASE
+MARKER CARD PRECEDING NEW DATA CASE.
                                                        1LINE CONSTANTS
+REQUEST FOR LINE CONSTANTS
                                                        1LINE-PARAMETERS
 COMMENT CARD.                                          1C
 COMMENT CARD.                                          1C <-UNIT<-------------------------INOPT
                                                        1ENGLISH
+ENGLISH  UNITS AND P.B.A.'S FORMAT ARE USED
 COMMENT CARD.                                          1C .....^...............................^
 COMMENT CARD.                                          1C                I
 COMMENT CARD.                                          1C                X       R
 COMMENT CARD.                                          1C P    S       R T    G  E       D       H       V       V
 S     A     N N
 COMMENT CARD.                                          1C H    K       E Y    M  A   CM  I   M   O    M  T    M  M
 CM  E DEG L     A B
 COMMENT CARD.                                          1C A    I       S P    R  C   IN  A  FT   R   FT  O   FT  I
 IN  P     P     M U
 COMMENT CARD.                                          1C
 S<---N<------I-E<------T-------M<------I<------W<------D<------A<----H<----E-N
 COMMENT CARD.                                          1C       1         2         3         4         5         6
 7         8
 COMMENT CARD.                                          1C
 345678901234567890123456789012345678901234567890123456789012345678901234567890
                                                        1  0       3.4468 2  0.1740   0.349    -8.0   131.0   131.0
 COMMENT CARD.                                          1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                        1  0       3.4468 2  0.1740   0.349     8.0   131.0   131.0
 COMMENT CARD.                                          1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
```

```
                                                   1  1       0.0863 2  0.4848   1.212   -17.5    96.0    96.0
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                   1  2       0.0863 2  0.4848   1.212   -13.5    81.0    81.0
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                   1  3       0.0863 2  0.4848   1.212   -13.5   113.0   113.0
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
 COMMENT CARD.                                     1C
                                                   1BLANK END OF CONDUCTOR DATA
+BLANK CARD TERMINATING LINE DATA
 COMMENT CARD.                                     1C
 COMMENT CARD.                                     1C ---RHO<-----FREQ<--------->ICPRNT-IZPRNT-C<----LEN
 <-PISM<-------> M F
                                                   1   90.00      50.0           111111 111111 1   100.0     10
                                                   1 1
+FREQUENCY CARD
 COMMENT CARD.                                     1C
 .....^.........^XXXXXXXXXXXX^^^^^^X^^^^^^^X^.......^X^^^^^^XXXXXXXXX.^.^
 COMMENT CARD.                                     1C
                                                   1.NODES              SEN_A     REC_A     SEN_B     REC_B
                                                   SEN_C     REC_C
+OPTIONAL NODE NAMES
 COMMENT CARD.                                     1C
 XXXXXXXXXXXXXXXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^
                                                   1BLANK CARD ENDIND FREQUENCY DATA
+BLANK CARD TERMINATING LINE PARAMETERS


 ////////////////////////////////////////////////////////////////////////////////

 ==========================      LINE-PARAMETERS    =============================

 ////////////////////////////////////////////////////////////////////////////////
```

BRITISH UNITS ARE USED.

RECORD OF SORTED INPUT DATA
(UNITS ARE THE SAME AS FOR INPUT)

PHASE	NUMBER	SKIN	DC RESIST	TYPE	PARAMETER	DIAMETER	X-COORD	Y-COORD	VREAL	VIMAGINARY
1	1	.0000	.08630	2	.48480	1.21200	-17.500	96.000	.000	.000
2	2	.0000	.08630	2	.48480	1.21200	-13.500	81.000	.000	.000
3	3	.0000	.08630	2	.48480	1.21200	-13.500	113.000	.000	.000
4	0	.0000	3.44680	2	.17400	.34900	-8.000	131.000	.000	.000
5	0	.0000	3.44680	2	.17400	.34900	8.000	131.000	.000	.000

==

FOLLOWING MATRICES ARE FOR EARTH RESISTIVITY= 90.00 OHM-M AND FREQUENCY= 50.00 HZ. CORRECTION FACTOR= .000001
EARTH WIRES WILL BE SEGMENTED

---> P INVERTED CAPACITANCE MATRIX (MILE/F)
+ FOR THE SYSTEM OF PHYSICAL CONDUCTORS
ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

1	9.20708E+07				
2	2.71859E+07	9.01731E+07			
3	2.77260E+07	2.01283E+07	9.38918E+07		
4	2.04947E+07	1.60712E+07	2.86202E+07	1.09448E+08	
5	1.85740E+07	1.52443E+07	2.42081E+07	3.12471E+07	1.09448E+08

---> C CAPACITANCE MATRIX (F/MILE)
+ FOR THE SYSTEM OF PHYSICAL CONDUCTORS
ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

```
   1   1.29476E-08

   2  -2.97723E-09   1.25512E-08

   3  -2.63523E-09  -1.40731E-09   1.28041E-08

   4  -1.04052E-09  -7.09328E-10  -2.20274E-09   1.06632E-08

   5  -9.02680E-10  -7.29132E-10  -1.55994E-09  -2.28173E-09   1.03880E-08


 ---> P      INVERTED CAPACITANCE MATRIX (MILE/F)
+      E                                              FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   8.66228E+07

   2   2.28278E+07   8.66837E+07

   3   2.03370E+07   1.42257E+07   8.38493E+07


 ---> P      INVERTED CAPACITANCE MATRIX (MILE/F)
+      S                                              FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   1.23979E+08
       0.00000E+00

   1   2.90431E+06  -4.45236E+06
       1.53725E+06   6.19862E+05

   2   2.90431E+06   6.65884E+07  -4.45236E+06
      -1.53725E+06   3.10441E-10  -6.19862E+05


 ---> C               CAPACITANCE MATRIX (F/MILE)
```

```
+      E                                        FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.29476E-08

   2  -2.97723E-09   1.25512E-08

   3  -2.63523E-09  -1.40731E-09   1.28041E-08



 ---> C              CAPACITANCE MATRIX (F/MILE)
+      S                                        FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   8.08777E-09
       0.00000E+00

   1  -3.76319E-10   1.02260E-09
      -1.71728E-10  -1.24456E-10

   2  -3.76319E-10   1.51075E-08   1.02260E-09
       1.71728E-10  -3.10193E-25   1.24456E-10



 ---> Z                  IMPEDANCE MATRIX (OHM/MILE)
+                                               FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.60539E-01
       1.13635E+00

   2   7.46087E-02   1.61283E-01
       5.34125E-01   1.13548E+00

   3   7.38261E-02   7.41904E-02   1.59719E-01
       5.23144E-01   4.61475E-01   1.13733E+00

   4   0.00000E+00   0.00000E+00   0.00000E+00   3.51938E+00
```

```
       0.00000E+00   0.00000E+00   0.00000E+00   1.24197E+00

   5   0.00000E+00   0.00000E+00   0.00000E+00   0.00000E+00   3.51938E+00
       0.00000E+00   0.00000E+00   0.00000E+00   0.00000E+00   1.24197E+00


 ---> Y          INVERTED IMPEDANCE MATRIX (S-MILE)
+                                                      FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                                       ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.80061E-01
      -1.24545E+00

   2  -6.47704E-02   1.62337E-01
       4.21673E-01  -1.17858E+00

   3  -5.95033E-02  -2.68520E-02   1.56942E-01
       4.00890E-01   2.86869E-01  -1.16363E+00

   4   0.00000E+00   0.00000E+00   0.00000E+00   2.52675E-01
       0.00000E+00   0.00000E+00   0.00000E+00  -8.91676E-02

   5   0.00000E+00   0.00000E+00   0.00000E+00   0.00000E+00   2.52675E-01
       0.00000E+00   0.00000E+00   0.00000E+00   0.00000E+00  -8.91676E-02


 ---> Z                   IMPEDANCE MATRIX (OHM/MILE)
+      E                                               FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                                       ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.60539E-01
       1.13635E+00

   2   7.46087E-02   1.61283E-01
       5.34125E-01   1.13548E+00

   3   7.38261E-02   7.41904E-02   1.59719E-01
       5.23144E-01   4.61475E-01   1.13733E+00
```

```
--------------------------------------------------------------------------------
                                 MODAL PARAMETERS AT FREQ =   5.00000E+01 HZ
--------------------------------------------------------------------------------


                                #####  MODAL WAVE QUANTITIES FROM Q COMPLEX  #####
                                            (Q ROTATED FOR GMODE = 0)

 MODE   RESISTANCE   REACTANCE  SUSCEPTANCE        SURGE IMPEDANCE(OHM)          VELOCITY   ATTENUATION
         OHM/MILE    OHM/MILE     S/MILE       REAL         IMAG       LOSSLESS   MILE/SEC   NEPER/MILE
   1    3.06519E-01 2.12764E+00 2.56544E-06 9.13032E+02-6.54305E+01 9.10684E+02 1.34123E+05 1.67858E-04
   2    8.61022E-02 5.83990E-01 5.06647E-06 3.40424E+02-2.49608E+01 3.39508E+02 1.82148E+05 1.26463E-04
   3    8.62127E-02 6.74420E-01 4.42436E-06 3.91221E+02-2.49040E+01 3.90427E+02 1.81500E+05 1.10184E-04


                       EIGENVECTOR MATRIX Q FOR CURRENT TRANSFORMATION I(PHASE)=Q*I(MODE)
                                            (Q ROTATED FOR GMODE = 0)

                                           REAL COMPONENTS, ROW BY ROW

 5.08449E-01 8.16221E-01 8.63913E-02
 5.65186E-01-4.67469E-01 6.75639E-01
 6.49582E-01-3.37931E-01-7.31235E-01

                                         IMAGINARY COMPONENTS, ROW BY ROW

 7.48755E-03 3.68049E-04-2.95342E-02
-5.18184E-03 1.89490E-02 1.75269E-02
-2.24298E-03-2.64502E-02 1.28423E-02


                                   Z-SURGE LOSSLESS IN PHASE COMPONENTS
                               (FROM Z-SURGE MODAL LOSSLESS AND WITH Q REAL)

 5.52854E+02
 2.03778E+02 5.53356E+02
 1.93154E+02 1.58714E+02 5.44491E+02


--------------------------------------------------------------------------------
```

```
 ---> Z                  IMPEDANCE MATRIX (OHM/MILE)
+      S                                            FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   3.08931E-01
       2.14888E+00

   1  -2.61348E-03  -6.87991E-03
       2.30458E-02  -4.47905E-02

   2   2.65687E-03   8.63055E-02   6.86935E-03
       2.16908E-02   6.30136E-01  -4.47911E-02

                              #####  SYMMETRICAL COMPONENTS WAVE QUANTITIES  #####
 SEQUENCE      SURGE IMPEDANCE       ATTENUATION   VELOCITY    WAVELENGTH   RESISTANCE    REACTANCE SUSCEPTANCE
         MAGNITUDE(OHM) ANGLE(DEGR.)   DB/MILE      MILES/SEC     MILES    OHM/MILE     OHM/MILE       S/MILE
   ZERO    9.24353E+02 -4.09050E+00  1.45518E-03  1.34104E+05  2.68208E+03  3.08931E-01  2.14888E+00  2.54085E-06
 POSITIVE  3.66069E+02 -3.89945E+00  1.02628E-03  1.81238E+05  3.62476E+03  8.63055E-02  6.30136E-01  4.74617E-06



 ---> Y          INVERTED IMPEDANCE MATRIX (S-MILE)
+      E                                            FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                              ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.80061E-01
      -1.24545E+00

   2  -6.47704E-02   1.62337E-01
       4.21673E-01  -1.17858E+00

   3  -5.95033E-02  -2.68520E-02   1.56942E-01
       4.00890E-01   2.86869E-01  -1.16363E+00


 ---> Y          INVERTED IMPEDANCE MATRIX (S-MILE)
```

```
+      S                                              FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   6.56961E-02
      -4.56266E-01

   1  -6.63652E-03   1.40136E-02
       1.67248E-02  -1.12323E-01

   2  -3.27229E-03   2.16822E-01   4.66473E-02
       1.66511E-02  -1.56570E+00  -1.03127E-01

 ================================================================================


                      :::::  PI-NOMINAL (APPROXIMATION) USING LUMPED LINE PARAMETERS  :::::


 MATRICES FOR LINE LENGTH = 1.000000E+02 MILES.
 THIS IS NOT AN EXACT EQUIVALENT PI-CIRCUIT, BUT AN APPROXIMATE PI-CIRCUIT


 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

                       PI-NOMINAL CIRCUIT FOR THE TOTAL LINE LENGTH WILL BE PUNCHED IN LU7


 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~



                                                  1BLANK CARD ENDING LINE CONSTANTS CASE
+BLANK CARD TERMINATING 'LINE CONSTANTS' CASES.

 CORE STORAGE FIGURES FOR PRECEDING DATA CASE NOW COMPLETED.  ---------------------------------------  PRESENT
 PROGRAM
 A VALUE OF  -9999 INDICATES DEFAULT WITH NO FIGURE AVAILABLE.                                         FIGURE
 LIMIT (NAME)
     SIZE LIST  1.   TOTAL STORAGE SPACE ALLOCATED FOR THE EMTP AUXILIARY PROGRAMS
```

```
    281040(LTLABL)
 TIMING FIGURES (DECIMAL) CHARACTERIZING CASE SOLUTION SPEED.  -------------------------------------    CP SEC
 I/O SEC   SUM SEC
                                                                                                   TOTALS       .060
                                                                                                   .000      .060


 SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
 DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.12
 FOR INFORMATION, CONSULT THE EMTP RULE BOOK.           PROGRAM VERSION="V2.0"
     LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
 --------------------------------------------------+------------------------------------------------------------
 ------------------
 DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
 CHARACTER BY CHARACTER.
                                                   0         1         2         3         4         5         6
                                                   7         8
                                                   0         0         0         0         0         0         0
                                                   0         0
 --------------------------------------------------+------------------------------------------------------------
 ------------------
                                                   1BLANK END OF EMTP DATA
+BLANK TERMINATION-OF-RUN CARD.
```

```
DONE READING DISK FILE INTO EMTP CACHE.   NUMCRD =   54  CARDS.
SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.16
FOR INFORMATION, CONSULT THE EMTP RULE BOOK.           PROGRAM VERSION="V2.0"
    LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
--------------------------------------------------+--------------------------------------------------------------------------------
DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
CHARACTER BY CHARACTER.
                                                  0         1         2         3         4         5         6         7         8
                                                  0         0         0         0         0         0         0         0         0
--------------------------------------------------+--------------------------------------------------------------------------------
COMMENT CARD.                                     1C SUPERHARM LINE MODEL TEST - CP-LINE EMTP SETUP - CASE 7D
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C NUMBER OF PHASE  CONDUCTORS ...: 3
COMMENT CARD.                                     1C NUMBER OF GROUND CONDUCTORS ...: 0
COMMENT CARD.                                     1C PHASE  CONDUCTORS TRANSPOSED ..: FALSE
COMMENT CARD.                                     1C GROUND CONDUCTORS SEGMENTED  ..: FALSE
COMMENT CARD.                                     1C FREQUENCY FOR CONSTANTS (HZ) ..: 60
COMMENT CARD.                                     1C EARTH RESTIVITY (OHM-METERS) ..: 100
COMMENT CARD.                                     1C UNITS FOR INPUT DATA ..........: METRIC (SI)
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C ID     DC RES.         GMR    DIAMETER     X COORD     Y COORD  NB     SPACING
COMMENT CARD.                                     1C      OHMS/KM          CM          CM      METERS      METERS              CM
COMMENT CARD.                                     1C --  ----------  ----------  ----------  ----------  ----------  --  ----------
COMMENT CARD.                                     1C  P    0.022145    0.799479     3.5103          -8          25   2          45
COMMENT CARD.                                     1C  P    0.022145    0.799459     3.5103           0          25   2          45
COMMENT CARD.                                     1C  P    0.022145    0.799479     3.5103           8          25   2          45
COMMENT CARD.                                     1C
COMMENT CARD.                                     1C
                                                  1BEGIN NEW DATA CASE
+MARKER CARD PRECEDING NEW DATA CASE.
                                                  1LINE CONSTANTS
```

```
+REQUEST FOR LINE CONSTANTS
                                                   1LINE-PARAMETERS
 COMMENT CARD.                                     1C
 COMMENT CARD.                                     1C <-UNIT<---------------------------INOPT
                                                   1METRIC                                 2
+METRIC   UNITS AND P.B.A.'S FORMAT ARE USED
 COMMENT CARD.                                     1C .....^...............................^
 COMMENT CARD.                                     1C                 I
 COMMENT CARD.                                     1C                 X       R
 COMMENT CARD.                                     1C P    S       R T    G  E       D       H       V       V
 S     A     N N
 COMMENT CARD.                                     1C H    K       E Y    M  A   CM  I   M   O    M  T    M  M
 CM  E DEG L     A B
 COMMENT CARD.                                     1C A    I       S P    R  C   IN  A  FT   R   FT  O   FT  I
 IN  P     P     M U
 COMMENT CARD.                                     1C
 S<---N<------I-E<------T-------M<------I<------W<------D<------A<----H<----E-N
 COMMENT CARD.                                     1C        1         2         3         4         5         6
 7         8
 COMMENT CARD.                                     1C
 345678901234567890123456789012345678901234567890123456789012345678901234567890
                                                   1  1     0.022145 2 .799479  3.5103    -8.0    25.0    25.0
                                                   45.0   0.0       2
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                   1  2     0.022145 2 .799479  3.5103     0.0    25.0    25.0
                                                   45.0   0.0       2
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
                                                   1  3     0.022145 2 .799479  3.5103     8.0    25.0    25.0
                                                   45.0   0.0       2
 COMMENT CARD.                                     1C
 ^....^.......^.^.......^.......^.......^.......^.......^.......^.....^.....^.^
 COMMENT CARD.                                     1C
                                                   1BLANK END OF CONDUCTOR DATA
+BLANK CARD TERMINATING LINE DATA
 COMMENT CARD.                                     1C
 COMMENT CARD.                                     1C ---RHO<-----FREQ<--------->ICPRNT-IZPRNT-C<----LEN
 <-PISM<-------> M F
                                                   1  100.00      60.0           111111 111111 1   100.0     00
                                                   1 1
+FREQUENCY CARD
```

```
 COMMENT CARD.                                           1C
 .....^.........^XXXXXXXXXXXX^^^^^^X^^^^^^^X^.......^X^^^^^^XXXXXXXXX.^.^
 COMMENT CARD.                                           1C
                                                         1.NODES              SEN_A     REC_A     SEN_B     REC_B
                                                         SEN_C     REC_C
+OPTIONAL NODE NAMES
 COMMENT CARD.                                           1C
 XXXXXXXXXXXXXXXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^XXXX.....^
                                                         1BLANK CARD ENDIND FREQUENCY DATA
+BLANK CARD TERMINATING LINE PARAMETERS
```

```
 ////////////////////////////////////////////////////////////////////////////////

 ==========================      LINE-PARAMETERS    =============================

 ////////////////////////////////////////////////////////////////////////////////
```

METRIC S.I. UNITS ARE USED.
(CM ASSUMED FOR COND. DIAMETER AND BUNDLE SPACING)

RECORD OF SORTED INPUT DATA
(UNITS ARE THE SAME AS FOR INPUT)

PHASE	NUMBER	SKIN	DC RESIST	TYPE	PARAMETER	DIAMETER	X-COORD	Y-COORD	VREAL	VIMAGINARY
1	1	.0000	.02215	2	.79948	3.51030	-8.225	25.000	.000	.000
2	2	.0000	.02215	2	.79948	3.51030	-.225	25.000	.000	.000
3	3	.0000	.02215	2	.79948	3.51030	7.775	25.000	.000	.000
4	1	.0000	.02215	2	.79948	3.51030	-7.775	25.000	.000	.000
5	2	.0000	.02215	2	.79948	3.51030	.225	25.000	.000	.000
6	3	.0000	.02215	2	.79948	3.51030	8.225	25.000	.000	.000

==

FOLLOWING MATRICES ARE FOR EARTH RESISTIVITY= 100.00 OHM-M AND FREQUENCY= 60.00 HZ. CORRECTION FACTOR= .000001

---> P INVERTED CAPACITANCE MATRIX (KM/F)
\+ FOR THE SYSTEM OF PHYSICAL CONDUCTORS
ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

1	1.42985E+08					
2	3.31680E+07	1.42985E+08				
3	2.13576E+07	3.31680E+07	1.42985E+08			
4	8.46729E+07	3.41841E+07	2.18240E+07	1.42985E+08		
5	3.22102E+07	8.46729E+07	3.41841E+07	3.31680E+07	1.42985E+08	
6	2.09065E+07	3.22102E+07	8.46729E+07	2.13576E+07	3.31680E+07	1.42985E+08

---> C CAPACITANCE MATRIX (F/KM)
\+ FOR THE SYSTEM OF PHYSICAL CONDUCTORS
ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

1	1.09807E-08					
2	-6.31822E-10	1.11395E-08				
3	-2.72482E-10	-6.17127E-10	1.10320E-08			
4	-6.14410E-09	-7.17982E-10	-2.90640E-10	1.10320E-08		
5	-5.49562E-10	-6.01266E-09	-7.17982E-10	-6.17127E-10	1.11395E-08	
6	-2.56634E-10	-5.49562E-10	-6.14410E-09	-2.72482E-10	-6.31822E-10	1.09807E-08

---> P INVERTED CAPACITANCE MATRIX (KM/F)

```
+      E                                              FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                                    ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   1.13819E+08

   2   3.31787E+07   1.13812E+08

   3   2.13698E+07   3.31787E+07   1.13819E+08



 ---> P      INVERTED CAPACITANCE MATRIX (KM/F)
+      S                                              FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                               ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                               ETC.

   0   1.72301E+08
       0.00000E+00

   1  -1.96706E+06   3.93738E+06
       3.40704E+06   6.81975E+06

   2  -1.96706E+06   8.45743E+07   3.93738E+06
      -3.40704E+06  -3.10441E-09  -6.81975E+06



 ---> C               CAPACITANCE MATRIX (F/KM)
+      E                                              FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                                    ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   9.72453E-09

   2  -2.51649E-09   1.02536E-08

   3  -1.09224E-09  -2.51649E-09   9.72453E-09



 ---> C               CAPACITANCE MATRIX (F/KM)
+      S                                              FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
```

```
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   5.81741E-09
       0.00000E+00

   1   1.49195E-10  -5.62933E-10
      -2.58413E-10  -9.75028E-10

   2   1.49195E-10   1.19426E-08  -5.62933E-10
       2.58413E-10   3.44659E-25   9.75028E-10


 ---> Z                   IMPEDANCE MATRIX (OHM/KM)
+                                                        FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   7.78976E-02
       8.76511E-01

   2   5.57464E-02   7.78976E-02
       3.55631E-01   8.76511E-01

   3   5.57280E-02   5.57464E-02   7.78976E-02
       3.03375E-01   3.55631E-01   8.76511E-01

   4   5.57526E-02   5.57471E-02   5.57293E-02   7.78976E-02
       5.72621E-01   3.59996E-01   3.05526E-01   8.76511E-01

   5   5.57457E-02   5.57526E-02   5.57471E-02   5.57464E-02   7.78976E-02
       3.51505E-01   5.72621E-01   3.59996E-01   3.55631E-01   8.76511E-01

   6   5.57266E-02   5.57457E-02   5.57526E-02   5.57280E-02   5.57464E-02   7.78976E-02
       3.01284E-01   3.51505E-01   5.72621E-01   3.03375E-01   3.55631E-01   8.76511E-01


 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+                                                        FOR THE SYSTEM OF PHYSICAL CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT
```

```
  1   1.38817E-01
     -2.10970E+00

  2  -5.04415E-04   1.39975E-01
      1.83043E-01  -2.15828E+00

  3   4.33943E-03  -3.09190E-04   1.39688E-01
      1.13558E-01   1.78512E-01  -2.12296E+00

  4  -9.93142E-02  -2.14973E-03   3.93977E-03   1.39688E-01
      1.15713E+00   1.96360E-01   1.15361E-01  -2.12296E+00

  5   9.23226E-04  -9.86355E-02  -2.14973E-03  -3.09190E-04   1.39975E-01
      1.67970E-01   1.11546E+00   1.96360E-01   1.78512E-01  -2.15828E+00

  6   4.69664E-03   9.23226E-04  -9.93142E-02   4.33943E-03  -5.04415E-04   1.38817E-01
      1.12155E-01   1.67970E-01   1.15713E+00   1.13558E-01   1.83043E-01  -2.10970E+00


 ---> Z                    IMPEDANCE MATRIX (OHM/KM)
+      E                                                FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                                ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

  1   6.68278E-02
      7.24529E-01

  2   5.57475E-02   6.68294E-02
      3.55676E-01   7.24507E-01

  3   5.57258E-02   5.57475E-02   6.68278E-02
      3.03420E-01   3.55676E-01   7.24529E-01


 --------------------------------------------------------------------------------
                                   MODAL PARAMETERS AT FREQ =   6.00000E+01 HZ
 --------------------------------------------------------------------------------

                                #####  MODAL WAVE QUANTITIES FROM Q COMPLEX  #####
```

```
                                    (Q ROTATED FOR GMODE = 0)

 MODE   RESISTANCE   REACTANCE  SUSCEPTANCE       SURGE IMPEDANCE(OHM)          VELOCITY   ATTENUATION
          OHM/KM      OHM/KM      S/KM          REAL        IMAG      LOSSLESS     KM/SEC     NEPER/KM
   1    1.77340E-01 1.39123E+00 2.20585E-06 7.95773E+02-5.05140E+01 7.94168E+02 2.14766E+05 1.11426E-04
   2    1.10747E-02 3.51503E-01 4.91399E-06 2.67486E+02-4.21276E+00 2.67453E+02 2.86811E+05 2.07015E-05
   3    1.11019E-02 4.21110E-01 4.07782E-06 3.21382E+02-4.23565E+00 3.21354E+02 2.87661E+05 1.72722E-05


                       EIGENVECTOR MATRIX Q FOR CURRENT TRANSFORMATION I(PHASE)=Q*I(MODE)
                                    (Q ROTATED FOR GMODE = 0)

                                   REAL COMPONENTS, ROW BY ROW

 6.06704E-01 4.07300E-01 7.07107E-01
 5.13630E-01-8.17442E-01 1.26283E-14
 6.06704E-01 4.07300E-01-7.07107E-01

                                  IMAGINARY COMPONENTS, ROW BY ROW

-8.04001E-04 5.37601E-04-3.38358E-15
 1.61361E-03 4.55128E-04 6.99862E-15
-8.04001E-04 5.37601E-04-3.39659E-15


                                Z-SURGE LOSSLESS IN PHASE COMPONENTS
                             (FROM Z-SURGE MODAL LOSSLESS AND WITH Q REAL)

 4.62965E+02
 1.82077E+02 4.62950E+02
 1.41611E+02 1.82077E+02 4.62965E+02



--------------------------------------------------------------------------------



---> Z                      IMPEDANCE MATRIX (OHM/KM)
+      S                                                FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                            ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
```

```
                          ETC.

   0   1.78309E-01
       1.40104E+00

   1  -1.50827E-02  -3.01698E-02
      -8.69902E-03   1.74346E-02

   2   1.50749E-02   1.10880E-02   3.01837E-02
      -8.71245E-03   3.86264E-01   1.74105E-02


                                 #####  SYMMETRICAL COMPONENTS WAVE QUANTITIES  #####
 SEQUENCE      SURGE IMPEDANCE       ATTENUATION   VELOCITY    WAVELENGTH   RESISTANCE    REACTANCE SUSCEPTANCE
         MAGNITUDE(OHM) ANGLE(DEGR.)   DB/KM          KM/S           KM    OHM/KM       OHM/KM        S/KM
   ZERO    8.02489E+02 -3.62650E+00  9.66916E-04  2.14636E+05  3.57726E+03  1.78309E-01  1.40104E+00  2.19311E-06
 POSITIVE  2.92965E+02 -8.22136E-01  1.64387E-04  2.85844E+05  4.76406E+03  1.10880E-02  3.86264E-01  4.50227E-06



 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+      E                                                FOR THE SYSTEM OF EQUIVALENT PHASE CONDUCTORS
                               ROWS AND COLUMNS PROCEED IN SAME ORDER AS SORTED INPUT

   1   7.98768E-02
      -1.91840E+00

   2  -2.04011E-03   8.26781E-02
       7.25884E-01  -2.08564E+00

   3   1.73153E-02  -2.04011E-03   7.98768E-02
       4.54632E-01   7.25884E-01  -1.91840E+00



 ---> Y          INVERTED IMPEDANCE MATRIX (S-KM)
+      S                                                FOR THE SYMMETRICAL COMPONENTS OF THE EQUIVALENT PHASE
CONDUCTORS
                          ROWS PROCEED IN SEQUENCE 0,1,2, 0,1,2 ETC. AND COLUMNS PROCEED IN SEQUENCE 0,2,1, 0,2,1
                          ETC.

   0   8.96339E-02
```

```
     -7.03211E-01

  1  -2.72656E-02  -2.11805E-01
     -2.21135E-02   1.06308E-01

  2   3.27837E-02   7.63989E-02   1.97968E-01
     -1.25560E-02  -2.60961E+00   1.30275E-01

 ================================================================================

                     :::::  PI-NOMINAL (APPROXIMATION) USING LUMPED LINE PARAMETERS  :::::

 MATRICES FOR LINE LENGTH = 1.000000E+02   KM
 THIS IS NOT AN EXACT EQUIVALENT PI-CIRCUIT, BUT AN APPROXIMATE PI-CIRCUIT

 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

                      PI-NOMINAL CIRCUIT FOR THE TOTAL LINE LENGTH WILL BE PUNCHED IN LU7

 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

                                                    1BLANK CARD ENDING LINE CONSTANTS CASE
+BLANK CARD TERMINATING 'LINE CONSTANTS' CASES.

 CORE STORAGE FIGURES FOR PRECEDING DATA CASE NOW COMPLETED.  ---------------------------------------  PRESENT
 PROGRAM
 A VALUE OF  -9999 INDICATES DEFAULT WITH NO FIGURE AVAILABLE.                                          FIGURE
 LIMIT (NAME)
     SIZE LIST  1.   TOTAL STORAGE SPACE ALLOCATED FOR THE EMTP AUXILIARY PROGRAMS
     281040(LTLABL)
 TIMING FIGURES (DECIMAL) CHARACTERIZING CASE SOLUTION SPEED.  -------------------------------------    CP SEC
 I/O SEC   SUM SEC
                                                                                              TOTALS       .000
                                                                                              .000      .000
```

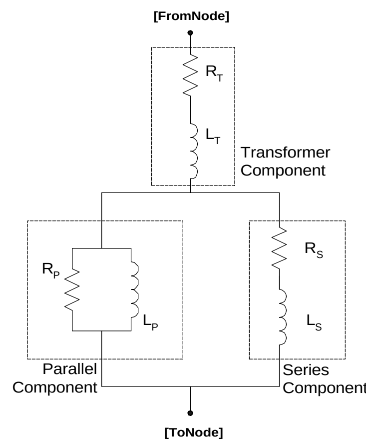
```
SUPPORTING AUXILIARY ROUTINES FOR EMTP - DCG/EPRI VERSION 2.0 DEC VAX TRANSLATION
DISTRIBUTED BY EPRI SOFTWARE CENTRE. RUN DATE (MM/DD/YY) AND TIME (HH.MM.SS.)= 06/19/98  09.26.16
FOR INFORMATION, CONSULT THE EMTP RULE BOOK.           PROGRAM VERSION="V2.0"
    LENGTH OF /LABEL/ EQUALS  281040 INTEGER WORDS, LENGTH OF INPUT DATA FILE EQUALS  2000 CARDS.
--------------------------------------------------+------------------------------------------------------------------
------------------
DESCRIPTIVE INTERPRETATION OF NEW-CASE INPUT DATA 1 INPUT DATA CARD IMAGES PRINTED BELOW, ALL 80 COLUMNS,
CHARACTER BY CHARACTER.
                                                  0         1         2         3         4         5         6
                                                  7         8
                                                  0         0         0         0         0         0         0
                                                  0         0
--------------------------------------------------+------------------------------------------------------------------
------------------
                                                  1BLANK END OF EMTP DATA
+BLANK TERMINATION-OF-RUN CARD.
```

SuperHarm Benchmarking - LINEARLOAD Modeling Equations

Electrotek Concepts - 6/22/98, TEG

This document illustrates the equations that define the operation of the LINEARLOAD model.

LINEARLOAD Model Verification:



Case 8b: - LinearLoad Model Verification

```
User defined values for the linear load model:

        kVA: Load kVA
         kV: System kV
         DF: Displacement Factor in per unit
    %Series: Percent series load
  %Parallel: Percent parallel load
```

Model Parameters

$KVA := 1.0$ $KV := 1.0$ $DF := 0.60$ $f := 60$ $DFang := acos(DF)\cdot\frac{180}{\pi}$

$\%Series := 50$ $\%Parallel := 100 - \%Series$ $\%Parallel = 50$ $DFang = 53.13$

$\omega := 2\cdot\pi\cdot f$ $\omega = 376.991$ $Zbase := \frac{1000\cdot KV^2}{KVA}$ $Zbase = 1000$ $h := 1$

Calculation of Series Components:

$KVAs := KVA\cdot\frac{\%Series}{100}$ $KVAs = 0.5$

$Ps := KVAs\cdot DF\cdot 1000$ $Ps = 300$

$Qs := KVAs\cdot 1000\cdot\sin(acos(DF))$ $Qs = 400$

$Zbs := \frac{1000\cdot KV^2}{KVAs}$ $Zbs = 2000$

$Rs := Zbs\cdot DF$

$Rs = 1200$

$Xs(h) := h\cdot Zbs\cdot\sin(acos(DF))$

$Xs(h) = 1600$

$Ls(h) := \frac{Xs(h)}{\omega}$ $Ls(h) = 4.244$

$Zs(h) := Rs + j\cdot\omega\cdot Ls(h)$

$Zs(h) = 1.2\bullet 10^3 + 1.6\bullet 10^3 i$

Calculation of Parallel Components:

$KVAp := KVA\cdot\frac{\%Parallel}{100}$ $KVAp = 0.5$

$Pp := KVAp\cdot DF\cdot 1000$

$Pp = 300$

$Qp := KVAp\cdot 1000\cdot\sin(acos(DF))$

$Qp = 400$

$Rp := \frac{KV^2\cdot 1000^2}{Pp}$ $Rp = 3333.333$

$Xp(h) := \frac{(KV)^2\cdot 1000^2\cdot h}{Qp}$ $Xp(h) = 2500$

$Lp(h) := \frac{Xp(h)}{\omega}$ $Lp(h) = 6.631$

$Zp1(h) := Rp + 0j$ $Zp2(h) := 0 + j\cdot\omega\cdot Lp(h$

$Zp(h) := \frac{Zp1(h)\cdot Zp2(h)}{(Zp1(h) + Zp2(h))}$

$|Zs(h)| = 2000 \quad arg(Zs(h)) \cdot \frac{180}{\pi} = 53.13$ $\qquad |Zp(h)| = 2000 \quad arg(Zp(h)) \cdot \frac{180}{\pi} = 53.13$

Calculation of combination of series and parallel components:

$Zeq(h) := \frac{Zs(h) \cdot Zp(h)}{Zs(h) + Zp(h)}$ $\qquad Zeq(h) = 600 + 800i$ $\qquad |Zeq(h)| = 1 \cdot 10^3$

$Req(h) := Re(Zeq(h))$ $\qquad Xeq(h) := Im(Zeq(h))$ $\qquad arg(Zeq(h)) \cdot \frac{180}{\pi} = 53.13$

$Ieq(h) := \frac{KV \cdot 1000}{Zeq(h)}$ $\quad Ieq(h) = 0.6 - 0.8i$

$Imag(h) := |Ieq(h)|$ $\qquad Iang(h) := \frac{180}{\pi} \cdot arg(Ieq(h))$

$Imag(h) = 1$ $\qquad Iang(h) = -53.13$

Shift Flow by 180 to correspond to Ip.Ground

$IpGround(h) := 180 + Iang(h)$

$IpGround(h) = 126.87$

SuperHarm Input/Output:

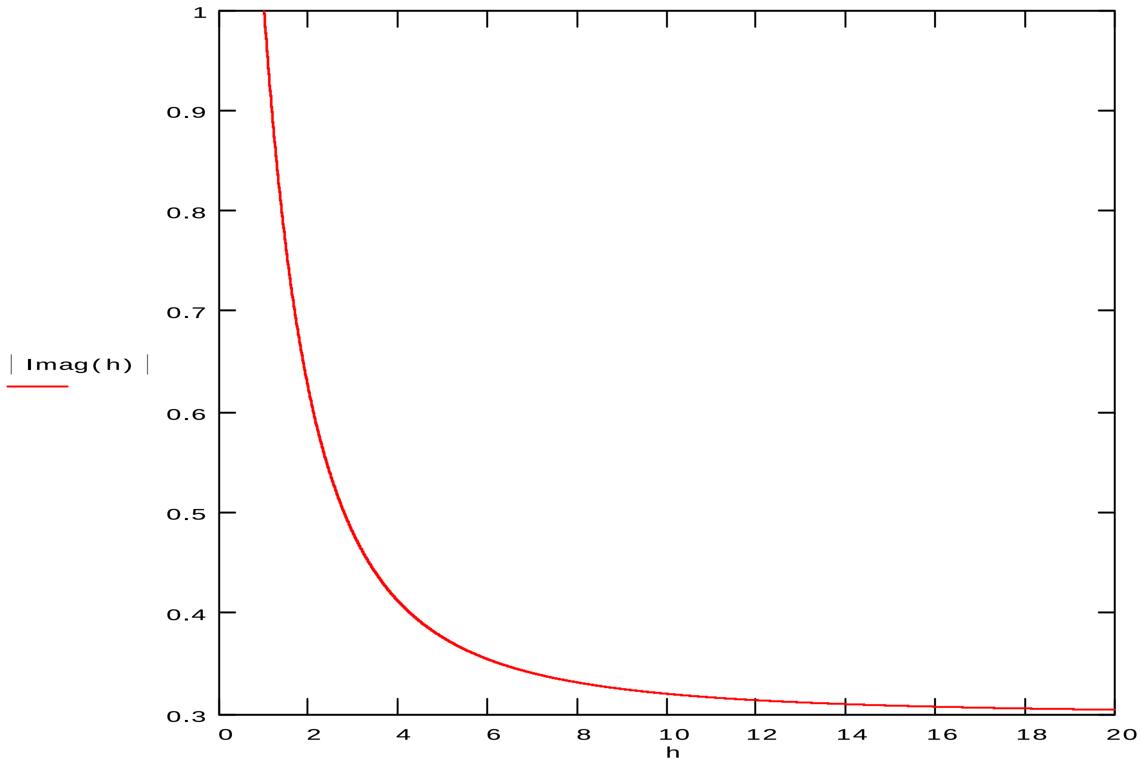
```
compute currents through 50% series load & 50% parallel load
at h = 1, 2, 3.  both loads are 1 kV, 1 kVA 60% dpf leading
____________________________________________________________________
df    kv    kva         P1       Q1      Rp      Xp      Rs      Xs
0.6    1     1        600.00  -800.00  3333.3 2500.0  1200.0  1600.0

CALCULATED CURRENTS (50% series, 50% parallel)
h   Rs      Xs      |Zs|     Rp       Xp     |Zp|   |Zeq|     |Isp|
1  1200  -1600.0  2000.0  3333.3  -2500.0  2000.0  1000.0    1.0000
2  1200  -3200.0  3417.6  3333.3  -5000.0  2773.5  1607.8    0.6219
3  1200  -4800.0  4947.7  3333.3  -7500.0  3046.0  2083.6    0.4799


Name      Freq       Mag              Angle (Frequency Domain Table)
Ip.Ground    1       1                126.9
Ip.Ground    2       0.621972         130.4
Ip.Ground    3       0.479924         136.7
```

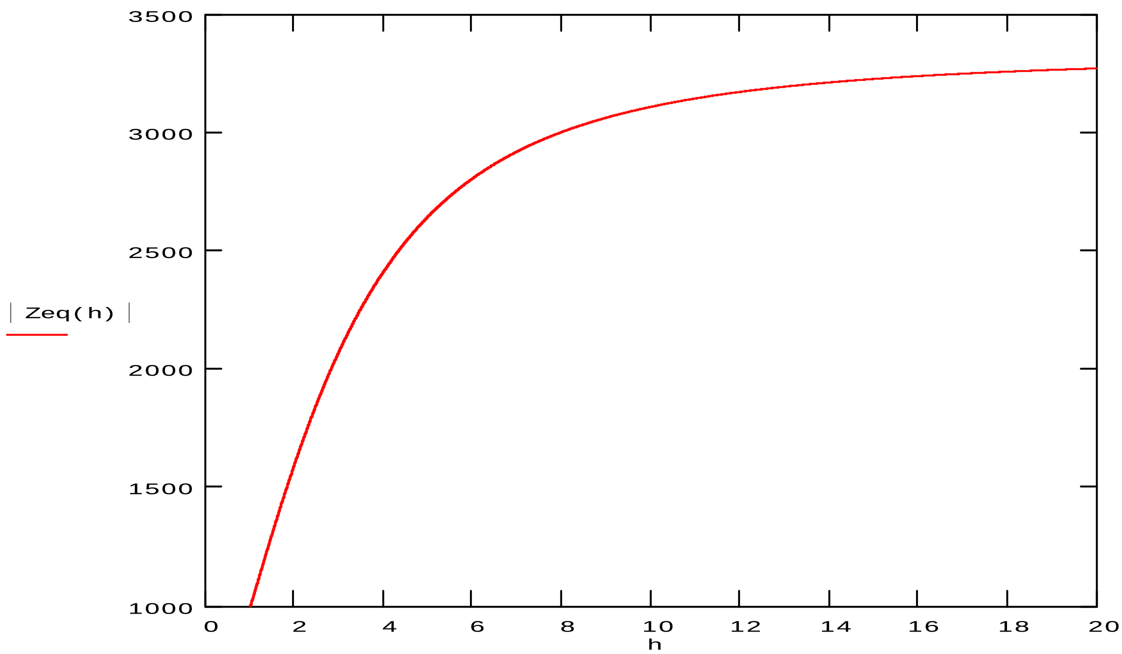
$h := 1..3$

h	Xs(h)	Xp(h)	|Zeq(h)|	Imag(h)	IpGround(h)
1	1600	2500	1000	1	126.87
2	3200	5000	1607.789	0.62197	130.355
3	4800	7500	2083.664	0.47992	136.655

Perform a frequency sweep of the solution:

h := 1, 1.01 .. 20



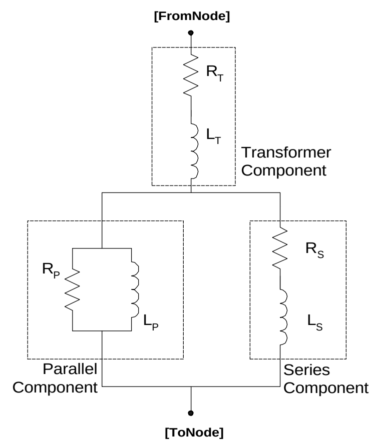


SuperHarm Benchmarking - LINEARLOAD Modeling Equations

Electrotek Concepts - 6/29/98, TEG

This document illustrates the equations that define the operation of the LINEARLOAD model.

LINEARLOAD Model Verification:



Case 8e: - LinearLoad Model Verification

User defined values for the linear load model:

```
       kVA: Load kVA
        kV: System kV
        DF: Displacement Factor in per unit
   %Series: Percent series load
 %Parallel: Percent parallel load
```

Model Parameters

[leading]

$KVA := 1.0$ $KV := 1.0$ $DF := 0.60$ $f := 60$ $DFang := acos(DF) \cdot \frac{180}{\pi}$

$\%Series := 50$ $\%Parallel := 100 - \%Series$ $\%Parallel = 50$ $DFang = 53.13$

$\omega := 2 \cdot \pi \cdot f$ $\omega = 376.991$ $Zbase := \frac{1000 \cdot KV^2}{KVA}$ $Zbase = 1000$ $h := 1$

Calculation of Series Components:

$KVAs := KVA \cdot \frac{\%Series}{100}$ $KVAs = 0.5$

$Ps := KVAs \cdot DF \cdot 1000$ $Ps = 300$

$Qs := KVAs \cdot 1000 \cdot \sin(acos(DF))$ $Qs = 400$

$Zbs := \frac{1000 \cdot KV^2}{KVAs}$ $Zbs = 2000$

$Rs := Zbs \cdot DF$

$Rs = 1200$

$Xs(h) := \frac{(-Zbs \cdot \sin(acos(DF)))}{h}$

$Xs(h) = -1600$

$Zs(h) := Rs + j \cdot Xs(h)$

$Zs(h) = 1200 - 1600i$

Calculation of Parallel Components:

$KVAp := KVA \cdot \frac{\%Parallel}{100}$ $KVAp = 0.5$

$Pp := KVAp \cdot DF \cdot 1000$

$Pp = 300$

$Qp := KVAp \cdot 1000 \cdot \sin(acos(DF))$

$Qp = 400$

$Rp := \frac{KV^2 \cdot 1000^2}{Pp}$ $Rp = 3333.333$

$Xp(h) := \frac{-(KV)^2 \cdot 1000^2}{Qp \cdot h}$ $Xp(h) = -2500$

$Zp1(h) := Rp + 0j$

$Zp2(h) := 0 + j \cdot Xp(h)$

$Zp(h) := \frac{Zp1(h) \cdot Zp2(h)}{(Zp1(h) + Zp2(h))}$

$|Zs(h)| = 2000 \quad arg(Zs(h))\cdot\frac{180}{\pi} = -53.13$ $\quad |Zp(h)| = 2000 \quad arg(Zp(h))\cdot\frac{180}{\pi} = -53.13$

Calculation of combination of series and parallel components:

$Zeq(h) := \frac{Zs(h)\cdot Zp(h)}{Zs(h) + Zp(h)}$ $\quad Zeq(h) = 600 - 800i$ $\quad |Zeq(h)| = 1\cdot 10^3$

$Req(h) := Re(Zeq(h))$ $\quad Xeq(h) := Im(Zeq(h))$ $\quad arg(Zeq(h))\cdot\frac{180}{\pi} = -53.13$

$Ieq(h) := \frac{KV\cdot 1000}{Zeq(h)}$ $\quad Ieq(h) = 0.6 + 0.8i$

$Imag(h) := |Ieq(h)|$ $\quad Iang(h) := \frac{180}{\pi}\cdot arg(Ieq(h))$

$Imag(h) = 1$ $\quad Iang(h) = 53.13$

SuperHarm Input/Output:

```
linearload name=load
           bus.a = node.a    bus.b = node.b     bus.c = node.c
           kv = 1.0  kva = 1.0       df = 0.6
           %parallel = 50  %series = 50
           leading=yes
```

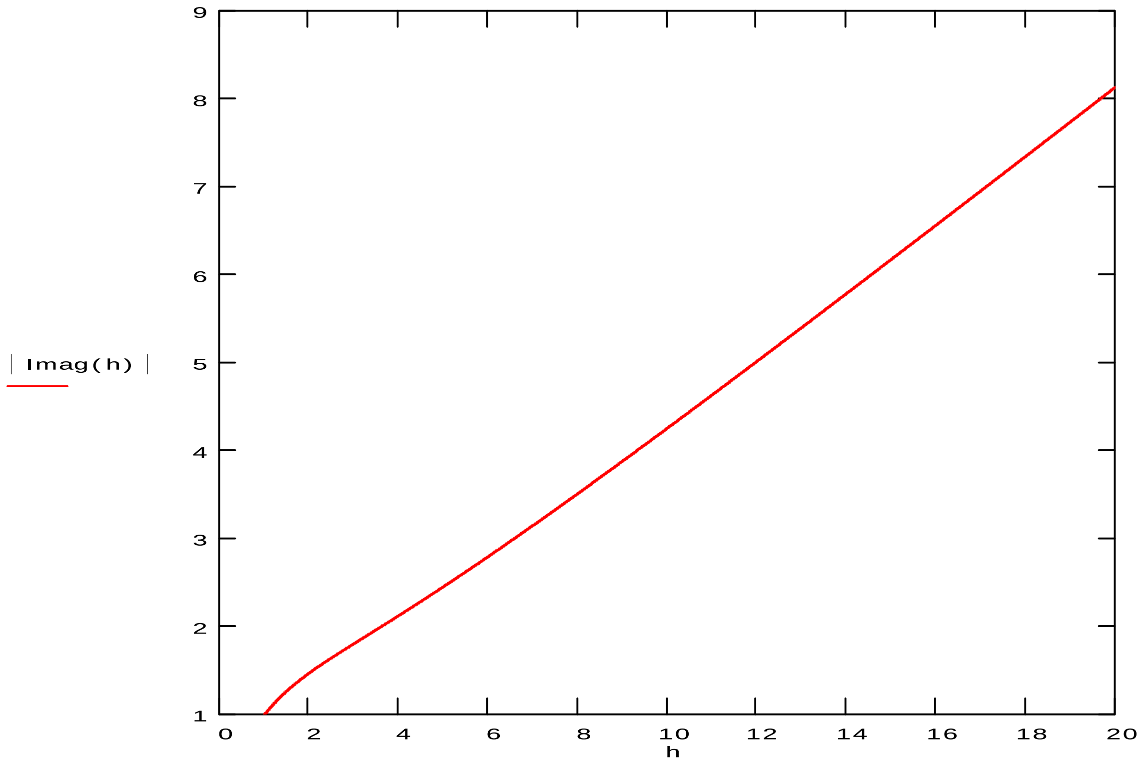
Name	Freq	Mag	Angle (Frequency Domain Table)
BR1A	1	1	-126.9
BR1A	2	1.47387	-126.5
BR1A	3	1.80823	-123.4

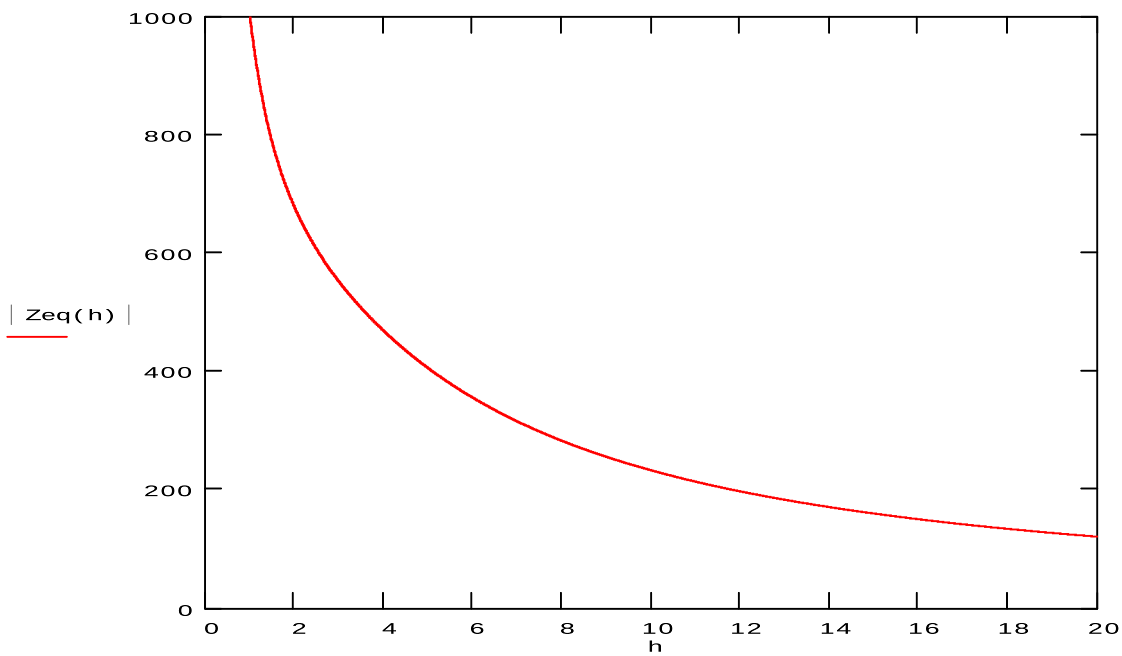
$h := 1..9$

h	Xs(h)	Xp(h)	|Zeq(h)|	Imag(h)	Iang(h) − 180
1	-1600	-2500	1000	1	-126.87
2	-800	-1250	678.484	1.47388	-126.511
3	-533.333	-833.333	553.027	1.80823	-123.418
4	-400	-625	470.1	2.1272	-119.578
5	-320	-500	407.062	2.45663	-116.028
6	-266.667	-416.667	357.25	2.79916	-113.009
7	-228.571	-357.143	317.174	3.15284	-110.5
8	-200	-312.5	284.474	3.51526	-108.421
9	-177.778	-277.778	257.44	3.8844	-106.688

Perform a frequency sweep of the solution:

h := 1, 1.01 .. 20





SuperHarm Benchmarking - MACHINE Modeling Equations

Electrotek Concepts - 6/12/98, TEG/EWG

This document illustrates the equations that define the operation of the MACHINE model.

MACHINE Model Verification:

```
! single phase test

VSOURCE NAME=SRC BUS=VSRC
MAG=277

BRANCH NAME=BR1 FROM=VSRC
TO=BUS1 X = 1.000

MACHINE NAME=M1 HP=100 KV=0.277
FROM=BUS1 DF=0.85
```

Cases 9a (single-phase model test):

Specified Parameters:

$HP := 100$ $V := 277$ $dPF := 0.85$ $f := 60$

Default Parameters:

$Eff := 0.90$ $Load := 1.00$

$h := 1$

$Zsrc(h) := 0 + i \cdot 1.00 \cdot h$

$Pm := (Eff \cdot Load \cdot HP \cdot 745.6)$ $Pm = 67104$

$Sm := \frac{Pm}{dPF}$ $Sm = 78945.882$

$Zbase := \frac{V^2}{Sm}$ $Zbase = 0.971919$

$kVAb := \frac{Sm}{1000}$ $kVAb = 78.946$

$Qm := \sqrt{(Sm^2 - Pm^2)}$ $Qm = 41587.324$

$Ym := \frac{(Pm - i \cdot Qm)}{V^2}$ $Ym = 0.875 - 0.542i$

$Ysrc(h) := \frac{1}{Zsrc(h)}$

$Zm := \frac{1}{Ym}$ $Zm = 0.826 + 0.512i$ $|Zm| = 0.972$ $arg(Zm) \cdot \frac{180}{\pi} = 31.788$

$Zm(h) := Re(Zm) + Im(Zm) \cdot h \cdot i$ $Zm(1) = 0.826 + 0.512i$

$Ztotal(h) := \frac{Zm(h) \cdot Zsrc(h)}{Zm(h) + Zsrc(h)}$ $Ztotal(1) = 0.278 + 0.491i$

$|Ztotal(1)| = 0.5641$ $arg(Ztotal(1)) \cdot \frac{180}{\pi} = 60.44$

$Xh := 0.17 \cdot (Zbase)$ $Xh = 0.165$ $Lh := \frac{Xh}{2 \cdot \pi \cdot f}$ $Lh = 0.00043828$ $Rh := \frac{Xh}{40}$

$Rh = 0.00413066$

X/R Constant="YES"

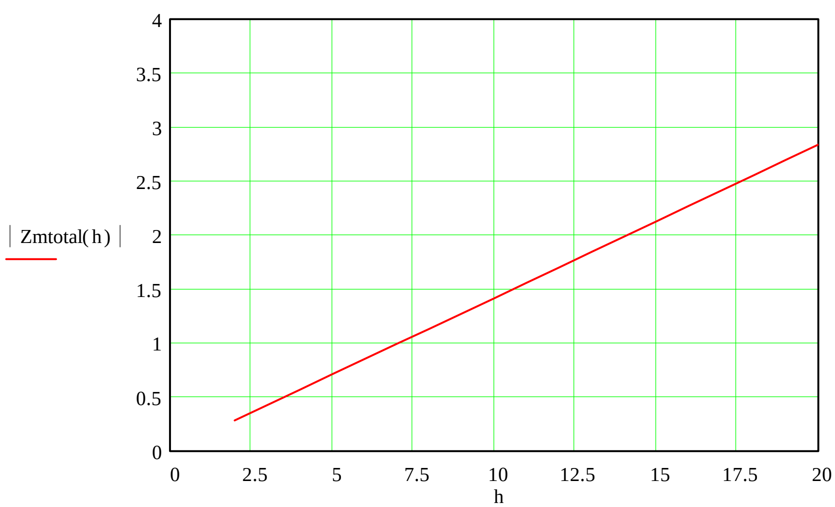
$Zmh(h) := Rh \cdot h + i \cdot h \cdot Xh$ $Ymh(h) := \frac{1}{Zmh(h)}$ $Zmh(2) = 0.008 + 0.33i$

$h := 2 .. 20$

$$Zmtotal(h) := \frac{Zmh(h) \cdot Zsrc(h)}{Zmh(h) + Zsrc(h)}$$

$Zmtotal(2) = 6.084 \cdot 10^{-3} + 0.284i$ $\quad |Zmtotal(2)| = 0.284$ $\quad arg(Zmtotal(2)) \cdot \frac{180}{\pi} = 88.771$

h	$Zmh(h)$	$\|Zmtotal(h)\|$	$arg(Zmtotal(h)) \cdot \frac{180}{\pi}$
2	$8.261 \cdot 10^{-3} + 0.33i$	0.28368	88.771
3	0.012 + 0.496i	0.42552	88.771
4	0.017 + 0.661i	0.56736	88.771
5	0.021 + 0.826i	0.7092	88.771
6	0.025 + 0.991i	0.85105	88.771
7	0.029 + 1.157i	0.99289	88.771
8	0.033 + 1.322i	1.13473	88.771
9	0.037 + 1.487i	1.27657	88.771
10	0.041 + 1.652i	1.41841	88.771
11	0.045 + 1.817i	1.56025	88.771
12	0.05 + 1.983i	1.70209	88.771
13	0.054 + 2.148i	1.84393	88.771
14	0.058 + 2.313i	1.98577	88.771
15	0.062 + 2.478i	2.12761	88.771
16	0.066 + 2.644i	2.26946	88.771
17	0.07 + 2.809i	2.4113	88.771
18	0.074 + 2.974i	2.55314	88.771
19	0.078 + 3.139i	2.69498	88.771
20	0.083 + 3.305i	2.83682	88.771



SuperHarm Benchmarking - MACHINE Modeling Equations

Electrotek Concepts - 6/12/98, TEG/EWG

This document illustrates the equations that define the operation of the MACHINE model.

MACHINE Model Verification:

! single phase test

VSOURCE NAME=SRC BUS=BUS1 MAG=277

MACHINE NAME=M1 HP=100 KV=0.277
KVABASE=100
FROM=BUS1 TO=BUS2 xrconstant=no
%LOAD=90 %EFF=95 DF=0.85 %xh=17.0

Cases 9b:

Parameters:

$HP := 100$ $V := 277$ $dPF := 0.85$ $f := 60$

$h := 1$

$Eff := 0.95$ $Load := 0.90$ $kVABase := 100$ $Zsrc(h) := 0 + i \cdot 0 \cdot h$

$Pm := (Eff \cdot Load \cdot HP \cdot 745.6)$ $Pm = 63748.8$

$Sm := \frac{Pm}{dPF}$ $Sm = 74998.588$

$Zbase := \frac{\left[1000 \cdot \left(\frac{V}{1000}\right)^2\right]}{kVABase}$

$Qm := \sqrt{\left(Sm^2 - Pm^2\right)}$ $Qm = 39507.958$

$Zbase = 0.76729$

$Ym := \frac{(Pm - i \cdot Qm)}{V^2}$ $Ym = 0.831 - 0.515i$

$Ysrc(h) := \frac{1}{Zsrc(h)}$

$Zm := \frac{1}{Ym}$ $Zm = 0.87 + 0.539i$ $|Zm| = 1.023$

$Zm(h) := Re(Zm) + Im(Zm) \cdot h \cdot i$ $Zm(1) = 0.87 + 0.539i$

$Zm(1) = 0.87 + 0.539i$ $|Zm(1)| = 1.023$ $arg(Zm(1)) \cdot \frac{180}{\pi} = 31.788$

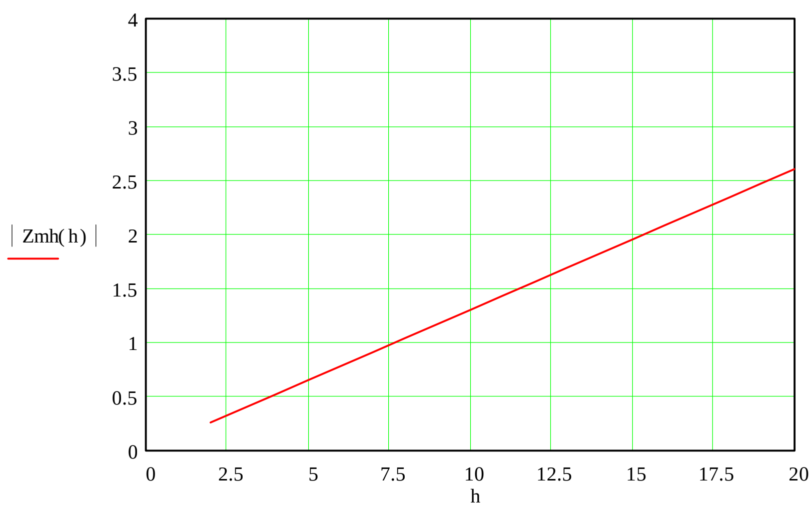
$Xh := 0.17 \cdot (Zbase)$ $Xh = 0.13$ $Rh := \frac{Xh}{40}$ $Rh = 3.261 \cdot 10^{-3}$

X/R Constant="NO"

$Zmh(h) := Rh + h \cdot (Xh \cdot i)$

$h := 2 .. 20$

h	$Zmh(h)$	$\lvert Zmh(h) \rvert$	$arg(Zmh(h)) \cdot \frac{180}{\pi}$
2	0.003 + 0.261i	0.2609	89.284
3	0.003 + 0.391i	0.39133	89.523
4	0.003 + 0.522i	0.52177	89.642
5	0.003 + 0.652i	0.6522	89.714
6	0.003 + 0.783i	0.78264	89.761
7	0.003 + 0.913i	0.91308	89.795
8	0.003 + 1.044i	1.04352	89.821
9	0.003 + 1.174i	1.17396	89.841
10	0.003 + 1.304i	1.3044	89.857
11	0.003 + 1.435i	1.43484	89.87
12	0.003 + 1.565i	1.56527	89.881
13	0.003 + 1.696i	1.69571	89.89
14	0.003 + 1.826i	1.82615	89.898
15	0.003 + 1.957i	1.95659	89.905
16	0.003 + 2.087i	2.08703	89.91
17	0.003 + 2.217i	2.21747	89.916
18	0.003 + 2.348i	2.34791	89.92
19	0.003 + 2.478i	2.47835	89.925
20	0.003 + 2.609i	2.60879	89.928



SuperHarm Benchmarking - MACHINE Modeling Equations

Electrotek Concepts - 6/11/98, TEG

This document illustrates the equations that define the operation of the MACHINE model.

MACHINE Model Verification:

```
machine
name=m1  hp=100      kv=0.277
bus.a=bus1a  bus.b=bus1b
bus.c=bus1c  connection=delta
 kvabase=78.946
df=0.85    %load=80  %eff=90
%rh=0.05   %xh=17.0
```

Cases 9e (single-phase equivalent):

Parameters:

$HP := 100$ $\quad V := 277$ $\quad dPF := 0.85$ $\quad f := 60$

$Eff := 0.90$ $\quad Load := 0.8$ $\quad kVABase := 78.946$

$h := 1$

$Zsrc(h) := 0 + i \cdot 1.00 \cdot h$

$Pm := (Eff \cdot Load \cdot HP \cdot 745.6)$ $\quad Pm = 53683.2$

$Sm := \frac{Pm}{dPF}$ $\quad Sm = 63156.706$

$Qm := \sqrt{(Sm^2 - Pm^2)}$ $\quad Qm = 33269.859$

$Zbase := \frac{\left[1000 \cdot \left(\frac{V}{1000}\right)^2\right]}{kVABase}$

$Zbase = 0.971918$

$Ym := \frac{(Pm - i \cdot Qm)}{V^2}$ $\quad Ym = 0.7 - 0.434i$

$Ysrc(h) := \frac{1}{Zsrc(h)}$

$Zm := \frac{1}{Ym}$ $\quad Zm = 1.033 + 0.64i$

$|Zm| = 1.215$ $\quad arg(Zm) \cdot \frac{180}{\pi} = 31.788$

$Zm(h) := Re(Zm) + Im(Zm) \cdot h \cdot i$ $\quad Zm(1) = 1.033 + 0.64i$

$Ztotal(h) := \frac{Zm(h) \cdot Zsrc(h)}{Zm(h) + Zsrc(h)}$ $\quad Ztotal(1) = 0.275 + 0.563i$

$|Ztotal(1)| = 0.62687$ $\quad arg(Ztotal(1)) \cdot \frac{180}{\pi} = 63.986$

$Xh := 0.17 \cdot (Zbase)$ $\quad Xh = 0.165$ $\quad Lh := \frac{Xh}{2 \cdot \pi \cdot f}$ $\quad Lh = 0.00043828$ $\quad Rh := \frac{Xh}{20}$

$Rh = 0.0082613$

X/R Constant="YES"

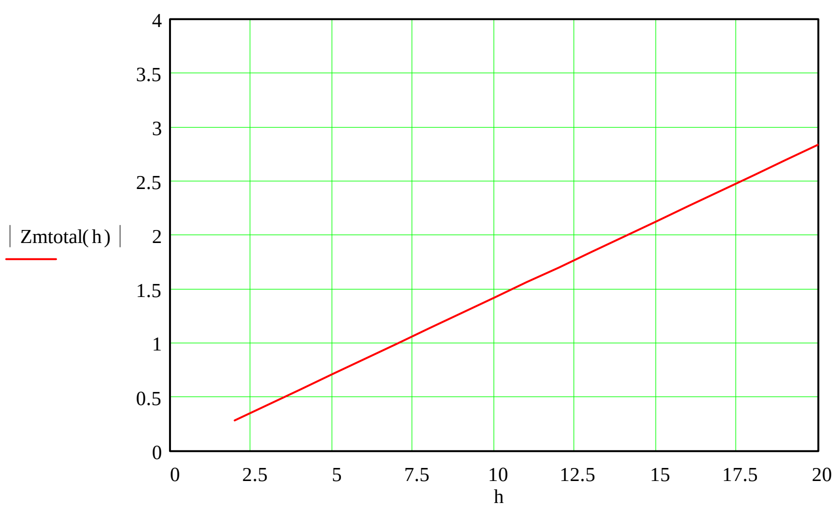
$Zmh(h) := Rh \cdot h + i \cdot h \cdot Xh$ $\quad Ymh(h) := \frac{1}{Zmh(h)}$ $\quad Zmh(2) = 0.017 + 0.33i$

$h := 2..20$

$$Zmtotal(h) := \frac{Zmh(h) \cdot Zsrc(h)}{Zmh(h) + Zsrc(h)}$$

$Zmtotal(2) = 0.012 + 0.284i$ $\quad |Zmtotal(2)| = 0.284$ $\quad arg(Zmtotal(2)) \cdot \frac{180}{\pi} = 87.544$

h	$Zmh(h)$	$\lvert Zmtotal(h)\rvert$	$arg(Zmtotal(h)) \cdot \frac{180}{\pi}$
2	0.017 + 0.33i	0.28394	87.544
3	0.025 + 0.496i	0.42591	87.544
4	0.033 + 0.661i	0.56788	87.544
5	0.041 + 0.826i	0.70985	87.544
6	0.05 + 0.991i	0.85183	87.544
7	0.058 + 1.157i	0.9938	87.544
8	0.066 + 1.322i	1.13577	87.544
9	0.074 + 1.487i	1.27774	87.544
10	0.083 + 1.652i	1.41971	87.544
11	0.091 + 1.817i	1.56168	87.544
12	0.099 + 1.983i	1.70365	87.544
13	0.107 + 2.148i	1.84562	87.544
14	0.116 + 2.313i	1.98759	87.544
15	0.124 + 2.478i	2.12956	87.544
16	0.132 + 2.644i	2.27153	87.544
17	0.14 + 2.809i	2.41351	87.544
18	0.149 + 2.974i	2.55548	87.544
19	0.157 + 3.139i	2.69745	87.544
20	0.165 + 3.305i	2.83942	87.544



SuperHarm Benchmarking - NONLINEARLOAD Modeling Equations

Electrotek Concepts - 6/17/98, TEG

This document illustrates the equations that define the operation of the NONLINEAR model.

NONLINEARLOAD Model Verification:

Case 10a:

```
VSOURCE NAME=VSRC BUS=SRC MAG=1000 ANG=0.0 FREQ=60

BRANCH  NAME=EQUIV FROM=SRC TO=NODE r=0.001

NONLINEARLOAD NAME=NL1 KV=1 KVA=1 DF=0.90
        FREQMULT = 1  BUS = NODE

TABLE={
      { 60,  100.0, 0.0 },
      { 180, 33.33, 0.0 },
      { 300, 20.00, 0.0 },
      { 420, 14.00, 0.0 },
      { 540, 11.11, 0.0 }
      }
```

$Va := 1000 + i \cdot 0$ $\quad Rs := 0.001$ $\quad |Va| = 1000$ $\quad arg(Va) \cdot \frac{180}{\pi} = 0$

$kV := 1$ $\quad kVA := 1$

$dPF := 0.90$ $\quad acos(dPF) \cdot \frac{180}{\pi} = 25.841933$

Fundamental Solution:

$Z1 := 1000 \cdot \frac{kV^2}{kVA}$ $\quad Z1 = 1000$ $\quad Z1ang := acos(dPF)$ $\quad acos(Z1ang) \cdot \frac{180}{\pi}$

$Zl := Z1 \cdot cos(Z1ang) + i \cdot Z1 \cdot sin(Z1ang)$ $\quad Zl = 900 + 435.889894i$ $\quad |Zl| = 1000$

$arg(Zl) \cdot \frac{180}{\pi} = 25.841933$

$Zckt := (Rs + i \cdot 0) + Zl$ $\quad Zckt = 900.001 + 435.889894i$

$I := \frac{Va}{Zckt}$ $\quad |I| = 0.999999$ $\quad arg(I) \cdot \frac{180}{\pi} = -25.841908$

$Vnl := Va - I \cdot (Rs + i \cdot 0)$ $\quad |Vnl| = 999.9991$ $\quad arg(Vnl) \cdot \frac{180}{\pi} = 0.000025$

Name	Frequency	Magnitude	Angle
(NL1)(NODE)	1	0.999999	-25.8
(NL1)(NODE)	3	0.3333	-77.5
(NL1)(NODE)	5	0.2	-129.2
(NL1)(NODE)	7	0.14	179.1
(NL1)(NODE)	9	0.1111	127.4
(NODE)()	1	999.999	0.0
(NODE)()	3	0.0003333	102.5
(NODE)()	5	0.0002	50.8
(NODE)()	7	0.00014	-0.9
(NODE)()	9	0.0001111	-52.6

$$\theta(h) := 0.0 + 1 \cdot h \cdot \arg(I) \cdot \frac{180}{\pi}$$

$$Ih1 := 100 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1} \qquad Ih1 = 0.999999 \qquad \theta(1) = -25.841908$$

$$Ih3 := 33.33 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1} \qquad Ih3 = 0.3333 \qquad \theta(3) = -77.525723$$

$$Ih5 := 20 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1} \qquad Ih5 = 0.2 \qquad \theta(5) = -129.209539$$

$$Ih7 := 14 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1} \qquad Ih7 = 0.14 \qquad \theta(7) + 360 = 179.106645$$

$$Ih9 := 11.11 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1} \qquad Ih9 = 0.1111 \qquad \theta(9) + 360 = 127.42283$$

$$I3 := Ih3 \cdot \cos\left(\theta(3) \cdot \frac{\pi}{180}\right) + i \cdot Ih3 \cdot \sin\left(\theta(3) \cdot \frac{\pi}{180}\right) \qquad |I3| = 0.3333 \qquad \arg(I3) \cdot \frac{180}{\pi} = -77.525723$$

$$Vnl3 := (0 + i \cdot 0) - (I3 \cdot (Rs + i \cdot 0)) \qquad |Vnl3| = 0.000333 \qquad \arg(Vnl3) \cdot \frac{180}{\pi} = 102.474277$$

$$I5 := Ih5 \cdot \cos\left(\theta(5) \cdot \frac{\pi}{180}\right) + \iota \cdot Ih5 \cdot \sin\left(\theta(5) \cdot \frac{\pi}{180}\right)$$

$$Vnl5 := (0 + \iota \cdot 0) - (I5 \cdot (Rs + \iota \cdot 0)) \qquad |Vnl5| = 0.0002 \qquad \arg(Vnl5) \cdot \frac{180}{\pi} = 50.790461$$

$$I7 := Ih7 \cdot \cos\left(\theta(7) \cdot \frac{\pi}{180}\right) + \iota \cdot Ih7 \cdot \sin\left(\theta(7) \cdot \frac{\pi}{180}\right)$$

$$Vnl7 := (0 + \iota \cdot 0) - (I7 \cdot (Rs + \iota \cdot 0)) \qquad |Vnl7| = 0.00014 \qquad \arg(Vnl7) \cdot \frac{180}{\pi} = -0.893355$$

$$I9 := Ih9 \cdot \cos\left(\theta(9) \cdot \frac{\pi}{180}\right) + \iota \cdot Ih9 \cdot \sin\left(\theta(9) \cdot \frac{\pi}{180}\right)$$

$$Vnl9 := (0 + \iota \cdot 0) - (I9 \cdot (Rs + \iota \cdot 0)) \qquad |Vnl9| = 0.0001111 \qquad \arg(Vnl9) \cdot \frac{180}{\pi} = -52.57717$$

SuperHarm Benchmarking - NONLINEARLOAD Modeling Equations

Electrotek Concepts - 6/17/98, TEG

This document illustrates the equations that define the operation of the NONLINEAR model.

NONLINEARLOAD Model Verification:

Case 10b:

```
VSOURCE NAME=VSRC BUS=SRC MAG=1000 ANG=0.0 FREQ=60

BRANCH  NAME=EQUIV FROM=SRC TO=NODE r=0.001

NONLINEARLOAD NAME=NL1 KV=1 KVA=1 DF=0.90
        FREQMULT = 1  BUS = NODE  LEADING=YES

TABLE={
      { 60,  100.0, 0.0 },
      { 180, 33.33, 0.0 },
      { 300, 20.00, 0.0 },
      { 420, 14.00, 0.0 },
      { 540, 11.11, 0.0 }
      }
```

$Va := 1000 + i \cdot 0$ $\quad Rs := 0.001$ $\quad |Va| = 1000$ $\quad arg(Va) \cdot \frac{180}{\pi} = 0$

$kV := 1$ $\quad kVA := 1$

$dPF := 0.90$ $\quad acos(dPF) \cdot \frac{180}{\pi} = 25.841933$

Fundamental Solution:

$Z1 := 1000 \cdot \frac{kV^2}{kVA}$ $\quad Z1 = 1000$ $\quad Z1ang := acos(dPF) \cdot -1$ $\quad acos(Z1ang) \cdot \frac{180}{\pi}$

$Zl := Z1 \cdot cos(Z1ang) + i \cdot Z1 \cdot sin(Z1ang)$ $\quad Zl = 900 - 435.889894i$ $\quad |Zl| = 1000$

$arg(Zl) \cdot \frac{180}{\pi} = -25.841933$

$Zckt := (Rs + i \cdot 0) + Zl$ $\quad Zckt = 900.001 - 435.889894i$

$I := \frac{Va}{Zckt}$ $\quad |I| = 0.999999$ $\quad arg(I) \cdot \frac{180}{\pi} = 25.841908$

$Vnl := Va - I \cdot (Rs + i \cdot 0)$ $\quad |Vnl| = 999.9991$ $\quad arg(Vnl) \cdot \frac{180}{\pi} = -0.000025$

Name	Frequency	Magnitude	Angle
(NL1)(NODE)	1	0.999999	25.8
(NL1)(NODE)	3	0.3333	77.5
(NL1)(NODE)	5	0.2	129.2
(NL1)(NODE)	7	0.14	-179.1
(NL1)(NODE)	9	0.1111	-127.4
(NODE)()	1	999.999	0.0
(NODE)()	3	0.0003333	-102.5
(NODE)()	5	0.0002	-50.8
(NODE)()	7	0.00014	0.9
(NODE)()	9	0.0001111	52.6

$\theta(h) := 0.0 + 1 \cdot h \cdot \arg(I) \cdot \frac{180}{\pi}$

$Ih1 := 100 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1}$ $\quad Ih1 = 0.999999 \quad \theta(1) = 25.841908$

$Ih3 := 33.33 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1}$ $\quad Ih3 = 0.3333 \quad \theta(3) = 77.525723$

$Ih5 := 20 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1}$ $\quad Ih5 = 0.2 \quad \theta(5) = 129.209539$

$Ih7 := 14 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1}$ $\quad Ih7 = 0.14 \quad \theta(7) - 360 = -179.106645$

$Ih9 := 11.11 \cdot \frac{1}{100} \cdot \left(\frac{kVA}{\frac{|Vnl|}{1000}} \right)^{-1}$ $\quad Ih9 = 0.1111 \quad \theta(9) - 360 = -127.42283$

$I3 := Ih3 \cdot \cos\left(\theta(3) \cdot \frac{\pi}{180}\right) + \iota \cdot Ih3 \cdot \sin\left(\theta(3) \cdot \frac{\pi}{180}\right)$ $\quad |I3| = 0.3333 \quad \arg(I3) \cdot \frac{180}{\pi} = 77.525723$

$Vnl3 := (0 + \iota \cdot 0) - (I3 \cdot (Rs + \iota \cdot 0))$ $\quad |Vnl3| = 0.000333 \quad \arg(Vnl3) \cdot \frac{180}{\pi} = -102.474277$

$$I5 := Ih5 \cdot \cos\left(\theta(5) \cdot \frac{\pi}{180}\right) + \iota \cdot Ih5 \cdot \sin\left(\theta(5) \cdot \frac{\pi}{180}\right)$$

$$Vnl5 := (0 + \iota \cdot 0) - (I5 \cdot (Rs + \iota \cdot 0)) \qquad |Vnl5| = 0.0002 \qquad \arg(Vnl5) \cdot \frac{180}{\pi} = -50.790461$$

$$I7 := Ih7 \cdot \cos\left(\theta(7) \cdot \frac{\pi}{180}\right) + \iota \cdot Ih7 \cdot \sin\left(\theta(7) \cdot \frac{\pi}{180}\right)$$

$$Vnl7 := (0 + \iota \cdot 0) - (I7 \cdot (Rs + \iota \cdot 0)) \qquad |Vnl7| = 0.00014 \qquad \arg(Vnl7) \cdot \frac{180}{\pi} = 0.893355$$

$$I9 := Ih9 \cdot \cos\left(\theta(9) \cdot \frac{\pi}{180}\right) + \iota \cdot Ih9 \cdot \sin\left(\theta(9) \cdot \frac{\pi}{180}\right)$$

$$Vnl9 := (0 + \iota \cdot 0) - (I9 \cdot (Rs + \iota \cdot 0)) \qquad |Vnl9| = 0.0001111 \qquad \arg(Vnl9) \cdot \frac{180}{\pi} = 52.57717$$

SuperHarm Benchmarking - PI Modeling Equations

Electrotek Concepts - 6/20/98, TEG/EWG

This document illustrates the equations that define the operation of the PI model.

PI Model Verification:

```
vsource name=vsrc
        bus=src    freq=60
        mag=1      ang=0

scan    name=scan1          bus=node
        fmin=60
        fmax=1200
        finc=60
        ang=0

pi      name=segment
        from=src      to=node
        x=0.6820
        cnf=12
        length = 100
```

Cases 11a, 11b:

Input Data (Units per Mile):

	Resistance (Ohms)	Reactance (Ohms @ 60 Hz)	Capacitance (Farads)
Positive Sequence	$R := 0.0341$	$X := 0.682$	$C := 12.0 \cdot 10^{-9}$
Frequency (Hertz)	$f := 60$		
Length (Miles)	$d := 100$		

Calculations:

$\omega := 2 \cdot \pi \cdot f$ $\quad L := \dfrac{X}{\omega}$ $\quad L = 1.809 \cdot 10^{-3}$ $\quad \omega = 376.991$

$z(\omega) := R + j \cdot \omega \cdot L$ $\quad z(\omega) = 0.034 + 0.682j$ $\quad z(2 \cdot \omega) = 0.034 + 1.364j$

$yc(\omega) := 0.0 + j \cdot \omega \cdot C$ $\quad yc(\omega) = 4.524 \cdot 10^{-6} j$ $\quad zl(\omega) := d \cdot z(\omega)$ $\quad |zl(\omega)| = 68.285$

$\gamma(\omega) := \sqrt{z(\omega) \cdot yc(\omega)}$ $\quad \gamma(\omega) = 4.39 \cdot 10^{-5} + 1.757 \cdot 10^{-3} j$ $\quad |\gamma(\omega)| = 1.758 \cdot 10^{-3}$

$d \cdot |\gamma(\omega)| = 0.17575977$

$Zc(\omega) := \sqrt{\dfrac{z(\omega)}{yc(\omega)}}$ $\quad Zc(\omega) = 388.393 - 9.704j$

$|Zc(\omega)| = 388.514$ $\quad \arg(Zc(\omega)) \cdot \dfrac{180}{\pi} = -1.431$

$\left| \dfrac{\sinh(d \cdot \gamma(\omega))}{d \cdot \gamma(\omega)} \right| = 0.995$

$zs(\omega) := d \cdot z(\omega) \dfrac{\sinh(d \cdot \gamma(\omega))}{d \cdot \gamma(\omega)}$ $\quad zs(\omega) = 3.375 + 67.851j$ $\quad |zs(\omega)| = 67.935$

$\arg(zs(\omega)) \cdot \dfrac{180}{\pi} = 87.152$

$ys(\omega) := \dfrac{1}{zs(\omega)}$

$$\left|\frac{\tanh\left(\frac{d\cdot\gamma(\omega)}{2}\right)}{\frac{d\cdot\gamma(\omega)}{2}}\right| = 1.003$$

$$ysh(\omega) := (yc(\omega)\cdot d)\cdot\frac{\tanh\left(\frac{d\cdot\gamma(\omega)}{2}\right)}{\frac{d\cdot\gamma(\omega)}{2}}$$

$ysh(\omega) = 5.852\cdot10^{-8} + 4.536\cdot10^{-4}j$ $|ysh(\omega)| = 4.5356\cdot10^{-4}$

$\arg(ysh(\omega))\cdot\frac{180}{\pi} = 89.993$

$zsh(\omega) := \frac{1}{ysh(\omega)}$ $|zs(\omega)| = 67.935$

$zs(\omega) = 3.375 + 67.851j$ $zsh(\omega) = 0.284 - 2.205\cdot10^{3}j$ $|zsh(\omega)| = 2.205\cdot10^{3}$

Cases 11a & 11b - Z//(Y/2):

$zsp(\omega) := 2.0\cdot\frac{1}{ysh(\omega)}$ $|zsp(\omega)| = 4409.598$

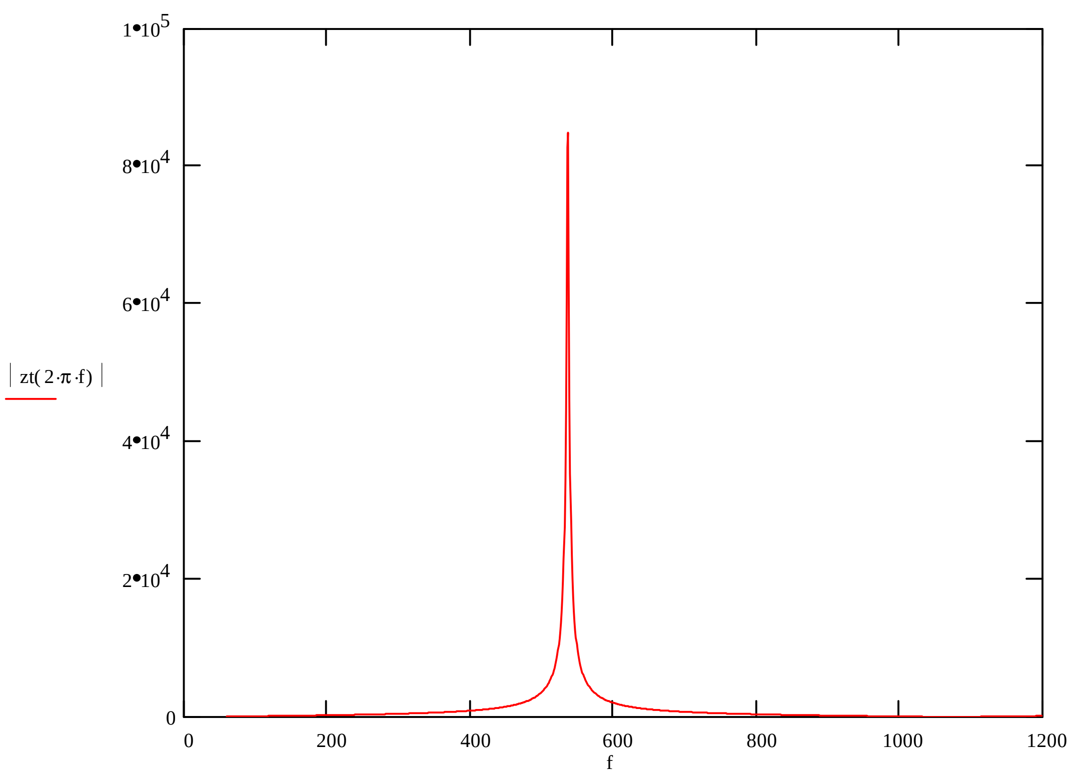
$zt(\omega) := \frac{zs(\omega)\cdot zsp(\omega)}{zs(\omega) + zsp(\omega)}$ $|zt(\omega)| = 68.9962267$ $\arg(zt(\omega))\cdot\frac{180}{\pi} = 87.108$

$f := 60, 120 .. 1200$

$\lvert Zc(2\cdot\pi\cdot f)\rvert$	$\lvert\gamma((2\ \pi\cdot f))\rvert$	$\left\lvert\frac{\tanh\left(\frac{d\cdot\gamma(2\cdot\pi\cdot f)}{2}\right)}{\frac{d\cdot\gamma(2\cdot\pi\cdot f)}{2}}\right\rvert$	$\left\lvert\frac{\sinh(d\cdot\gamma(2\cdot\pi\cdot f))}{d\cdot\gamma(2\cdot\pi\cdot f)}\right\rvert$
388.514	0.00176	1.002579	0.994866
388.333	0.00351	1.010413	0.979558
388.299	0.00527	1.023801	0.954359
388.287	0.00703	1.043275	0.919733
388.282	0.00878	1.069654	0.876314
388.279	0.01054	1.104141	0.824896
388.277	0.0123	1.148469	0.76641
388.276	0.01405	1.205155	0.701908
388.275	0.01581	1.277924	0.63254
388.274	0.01757	1.372456	0.559527
388.274	0.01932	1.497797	0.484135
388.274	0.02108	1.669197	0.407652
388.273	0.02283	1.91446	0.331356
388.273	0.02459	2.290067	0.256492
388.273	0.02635	2.930865	0.184243
388.273	0.0281	4.257783	0.115711
388.273	0.02986	8.591645	0.051899
388.273	0.03162	61.454979	$6.51055\cdot10^{-3}$
388.273	0.03337	6.101415	0.058299
388.273	0.03513	3.030139	0.103318

f	$\lvert zt(2\pi \cdot f) \rvert$	$\arg(zt(2 \cdot \pi \cdot f)) \cdot \frac{180}{\pi}$
60	68.99623	87.108
120	142.34703	88.505
180	225.93822	88.944
240	328.79081	89.132
300	468.02754	89.202
360	683.04762	89.176
420	1093.25431	88.998
480	2322.5476	88.274
540	35370.24135	-66.577
600	2066.10149	-88.757
660	1027.23169	-89.369
720	652.13794	-89.547
780	449.30328	-89.602
840	315.58477	-89.588
900	215.58083	-89.503
960	133.51937	-89.271
1020	60.90676	-88.441
1080	7.99392	77.6
1140	77.0139	88.606
1200	151.237	89.184

$f := 60, 61 .. 1200$



SuperHarm Benchmarking - PI3
Modeling Equations

Electrotek Concepts - 6/20/98, TEG/EWG

This document illustrates the equations that define the operation of the PI3 model.

PI3 Model Verification:

Cases 12a, 12b, 12c, & 12d:

Input Data (Units per Mile):

	Resistance (Ohms)	Reactance (Ohms @ 60 Hz)	Capacitance (Farads)
Zero Sequence	$R_0 := 0.3012$	$X_0 := 3.012$	$C_0 := 8.4 \cdot 10^{-9}$
Positive Sequence	$R_1 := 0.03410$	$X_1 := 0.682$	$C_1 := 12.0 \cdot 10^{-9}$
Frequency (Hertz)	$f := 60$		
Length (Miles)	$d := 100$		

Calculations:

$\omega := 2 \cdot \pi \cdot f$ $\quad L := \frac{X}{\omega}$ $\quad L = \begin{bmatrix} 7.98958 \cdot 10^{-3} \\ 1.80906 \cdot 10^{-3} \end{bmatrix}$

$z(\omega) := R + j \cdot \omega \cdot L$ $\quad z(\omega) = \begin{bmatrix} 0.3012 + 3.012j \\ 0.0341 + 0.682j \end{bmatrix}$

$yc(\omega) := 0.0 + j \cdot \omega \cdot C$ $\quad yc(\omega) = \begin{bmatrix} 3.16673 \cdot 10^{-6} j \\ 4.52389 \cdot 10^{-6} j \end{bmatrix}$

$\gamma(\omega) := \overrightarrow{\sqrt{z(\omega) \cdot yc(\omega)}}$ $\quad \gamma(\omega) = \begin{bmatrix} 1.54228 \cdot 10^{-4} + 3.09224 \cdot 10^{-3} j \\ 4.38988 \cdot 10^{-5} + 1.75705 \cdot 10^{-3} j \end{bmatrix}$

$Zc(\omega) := \overrightarrow{\sqrt{\frac{z(\omega)}{yc(\omega)}}}$ $\quad Zc(\omega) = \begin{bmatrix} 976.47949 - 48.70252j \\ 388.39318 - 9.70377j \end{bmatrix}$

$\overrightarrow{\sinh(\gamma(\omega) \cdot d)} = \begin{bmatrix} 0.01469 + 0.30436j \\ 4.32231 \cdot 10^{-3} + 0.1748j \end{bmatrix}$ $\quad \gamma(\omega) \cdot d = \begin{bmatrix} 0.01542 + 0.30922j \\ 4.38988 \cdot 10^{-3} + 0.1757j \end{bmatrix}$

Equivalent PI Circuit Elements

$$\mathrm{Zprime}(l,\omega) := \overrightarrow{z(\omega)\cdot\frac{\sinh(\gamma(\omega)\cdot l)}{\gamma(\omega)\cdot l}} \qquad \mathrm{Zprime}(d,\omega) = \begin{bmatrix} 0.29169 + 2.96482j \\ 0.03375 + 0.67851j \end{bmatrix}$$

$$\mathrm{YCprime}(l,\omega) := \overrightarrow{yc(\omega)\cdot\frac{\tanh\left(\frac{\gamma(\omega)\cdot l}{2}\right)}{\left(\frac{\gamma(\omega)\cdot l}{2}\right)}} \qquad \mathrm{YCprime}(d,\omega) = \begin{bmatrix} 2.56578\bullet 10^{-9} + 3.19214\bullet 10^{-6}j \\ 5.8517\bullet 10^{-10} + 4.53556\bullet 10^{-6}j \end{bmatrix}$$

Convert from sequence domain to phase domain:

$$\mathrm{Zps}(d,\omega) := \frac{\mathrm{Zprime}(d,\omega)_0 + 2\cdot\mathrm{Zprime}(d,\omega)_1}{3} \qquad \mathrm{Zps}(d,\omega) = 0.11973 + 1.44061j$$

$$\mathrm{Zpm}(d,\omega) := \frac{\mathrm{Zprime}(d,\omega)_0 - \mathrm{Zprime}(d,\omega)_1}{3} \qquad \mathrm{Zpm}(d,\omega) = 0.08598 + 0.7621j$$

$$\mathrm{YCps}(d,\omega) := \frac{\mathrm{YCprime}(d,\omega)_0 + 2\cdot\mathrm{YCprime}(d,\omega)_1}{3} \qquad \mathrm{YCps}(d,\omega) = 1.24537\bullet 10^{-9} + 4.08775\bullet 10^{-6}j$$

$$\mathrm{YCpm}(d,\omega) := \frac{\mathrm{YCprime}(d,\omega)_0 - \mathrm{YCprime}(d,\omega)_1}{3} \qquad \mathrm{YCpm}(d,\omega) = 6.60205\bullet 10^{-10} - 4.47808\bullet 10^{-7}j$$

$$\mathrm{YZp}(d,\omega) := \begin{bmatrix} \mathrm{Zps}(d,\omega) & \mathrm{Zpm}(d,\omega) & \mathrm{Zpm}(d,\omega) \\ \mathrm{Zpm}(d,\omega) & \mathrm{Zps}(d,\omega) & \mathrm{Zpm}(d,\omega) \\ \mathrm{Zpm}(d,\omega) & \mathrm{Zpm}(d,\omega) & \mathrm{Zps}(d,\omega) \end{bmatrix}^{-1} \qquad \mathrm{YCp}(d,\omega) := \begin{bmatrix} \mathrm{YCps}(d,\omega) & \mathrm{YCpm}(d,\omega) & \mathrm{YCpm}(d,\omega) \\ \mathrm{YCpm}(d,\omega) & \mathrm{YCps}(d,\omega) & \mathrm{YCpm}(d,\omega) \\ \mathrm{YCpm}(d,\omega) & \mathrm{YCpm}(d,\omega) & \mathrm{YCps}(d,\omega) \end{bmatrix}$$

$$\mathrm{YZCp}(d,w) := \mathrm{YZp}(d,\omega) + \mathrm{YCp}(d,\omega)$$

$$\mathrm{YZp}(d,\omega) = \begin{bmatrix} 0.05971 - 1.09148j & -0.01342 + 0.37871j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & 0.05971 - 1.09148j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & -0.01342 + 0.37871j & 0.05971 - 1.09148j \end{bmatrix}$$

$$\mathrm{YZCp}(d,\omega) = \begin{bmatrix} 0.05971 - 1.09147j & -0.01342 + 0.37871j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & 0.05971 - 1.09147j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & -0.01342 + 0.37871j & 0.05971 - 1.09147j \end{bmatrix}$$

$$\mathrm{yds}(d,\omega) := \mathrm{YZp}(d,\omega)_{0,0} \qquad \mathrm{ydm}(d,\omega) := \mathrm{YZp}(d,\omega)_{0,1}$$

$$\mathrm{yos}(d,\omega) := \mathrm{YZCp}(d,\omega)_{0,0} \qquad \mathrm{yom}(d,\omega) := \mathrm{YZCp}(d,\omega)_{0,1}$$

Build the primative admittance matrix

$$Y(d,\omega) := \begin{bmatrix} yds(d,\omega) & ydm(d,\omega) & ydm(d,\omega) & yos(d,\omega) & yom(d,\omega) & yom(d,\omega) \\ ydm(d,\omega) & yds(d,\omega) & ydm(d,\omega) & yom(d,\omega) & yos(d,\omega) & yom(d,\omega) \\ ydm(d,\omega) & ydm(d,\omega) & yds(d,\omega) & yom(d,\omega) & yom(d,\omega) & yos(d,\omega) \\ yos(d,\omega) & yom(d,\omega) & yom(d,\omega) & yds(d,\omega) & ydm(d,\omega) & ydm(d,\omega) \\ yom(d,\omega) & yos(d,\omega) & yom(d,\omega) & ydm(d,\omega) & yds(d,\omega) & ydm(d,\omega) \\ yom(d,\omega) & yom(d,\omega) & yos(d,\omega) & ydm(d,\omega) & ydm(d,\omega) & yds(d,\omega) \end{bmatrix}$$

Construct some transposition matricies to help extract
sub-matrices needed to solve for |H| in the equation above:

$$T1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad T2 := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T3 := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculate sub-matrices

$$F(d,\omega) := T3 \cdot Y(d,\omega) \cdot T2 \qquad D(d,\omega) := T1 \cdot Y(d,\omega) \cdot T2$$

$$F(d,\omega) = \begin{bmatrix} 0.05971 - 1.09148j & -0.01342 + 0.37871j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & 0.05971 - 1.09148j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & -0.01342 + 0.37871j & 0.05971 - 1.09148j \end{bmatrix}$$

$$D(d,\omega) = \begin{bmatrix} 0.05971 - 1.09147j & -0.01342 + 0.37871j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & 0.05971 - 1.09147j & -0.01342 + 0.37871j \\ -0.01342 + 0.37871j & -0.01342 + 0.37871j & 0.05971 - 1.09147j \end{bmatrix}$$

Calculate the driving point impedance:

$$Zdp(d,\omega) := F(d,\omega)^{(-1)}$$

$$Zdp(d,\omega) = \begin{bmatrix} 0.11973 + 1.44061j & 0.08598 + 0.7621j & 0.08598 + 0.7621j \\ 0.08598 + 0.7621j & 0.11973 + 1.44061j & 0.08598 + 0.7621j \\ 0.08598 + 0.7621j & 0.08598 + 0.7621j & 0.11973 + 1.44061j \end{bmatrix}$$

Build a driving voltage vector:

$$Vi := \begin{bmatrix} \left(\cos\left(0\cdot\frac{\pi}{180}\right) + j\cdot\sin\left(0\cdot\frac{\pi}{180}\right)\right)\cdot 1 \\ \left(\cos\left(-120\cdot\frac{\pi}{180}\right) + j\cdot\sin\left(-120\cdot\frac{\pi}{180}\right)\right)\cdot 1.1 \\ \left(\cos\left(120\cdot\frac{\pi}{180}\right) + j\cdot\sin\left(120\cdot\frac{\pi}{180}\right)\right)\cdot 1.05 \end{bmatrix} \qquad Vi = \begin{bmatrix} 1 \\ -0.55 - 0.95263j \\ -0.525 + 0.90933j \end{bmatrix}$$

Solve the equation:

$$B(d,\omega) := -D(d,\omega)\cdot Vi \qquad Vo(d,\omega) := \left(F(d,\omega)^{-1}\cdot B(d,\omega)\right) \qquad Vo(d,\omega) = \begin{bmatrix} -1 - 2.26015\bullet 10^{-7}j \\ 0.55 + 0.95263j \\ 0.525 - 0.90932j \end{bmatrix}$$

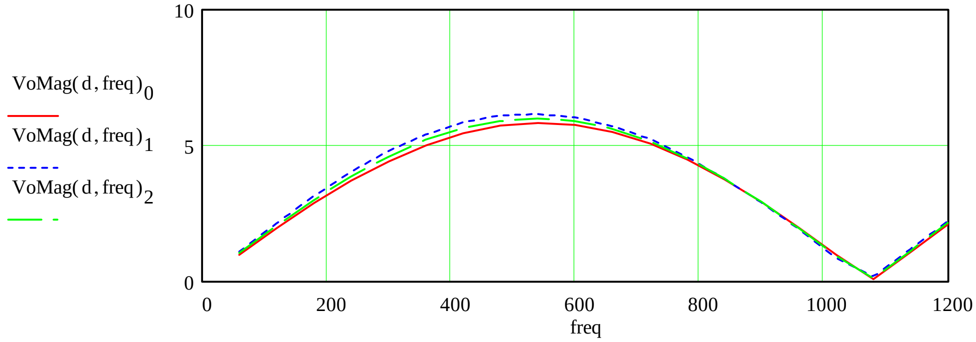
Convert to polar coordinates:

$$VoMag(d,f) := \overrightarrow{\left|Vo(d,2\cdot\pi\cdot f)\right|} \qquad VoAng(d,\omega) := \overrightarrow{\frac{180}{\pi}\cdot angle(Re(Vo(d,\omega)), Im(Vo(d,\omega)))}$$

$$VoMag(d,f) = \begin{bmatrix} 1 \\ 1.1 \\ 1.05 \end{bmatrix} \qquad VoAng(d,\omega) = \begin{bmatrix} 180.00001 \\ 60.00001 \\ 300 \end{bmatrix}$$

Perform a frequency sweep of the solution for recieving end voltage:

$$freq := 60, 120 .. 1200$$



Cases 12a... - Z//(Y/2):

Equivalent PI Circuit Elements

$$Zprime(d,\omega) := \overrightarrow{\left(z(\omega)\cdot\frac{\sinh(\gamma(\omega)\cdot d)}{\gamma(\omega)\cdot d}\right)}$$

$$Zprime(d,\omega) = \begin{bmatrix} 0.29169 + 2.96482j \\ 0.03375 + 0.67851j \end{bmatrix} \qquad d = 100$$

$$YCprime(d,\omega) := \overrightarrow{yc(\omega)\cdot\frac{\tanh\left(\frac{\gamma(\omega)\cdot d}{2}\right)}{\left(\frac{\gamma(\omega)\cdot d}{2}\right)}}$$

$$YCprime(d,\omega) = \begin{bmatrix} 2.56578\bullet10^{-9} + 3.19214\bullet10^{-6}j \\ 5.8517\bullet10^{-10} + 4.53556\bullet10^{-6}j \end{bmatrix}$$

$$Zspos(\omega) := (d\cdot Zprime(d,\omega))_1 \qquad Zspos(\omega) = 3.37501 + 67.85072j$$

$$Yppos(\omega) := (d\cdot YCprime(d,\omega))_1 \qquad Yppos(\omega) = 5.8517\bullet10^{-8} + 4.53556\bullet10^{-4}j$$

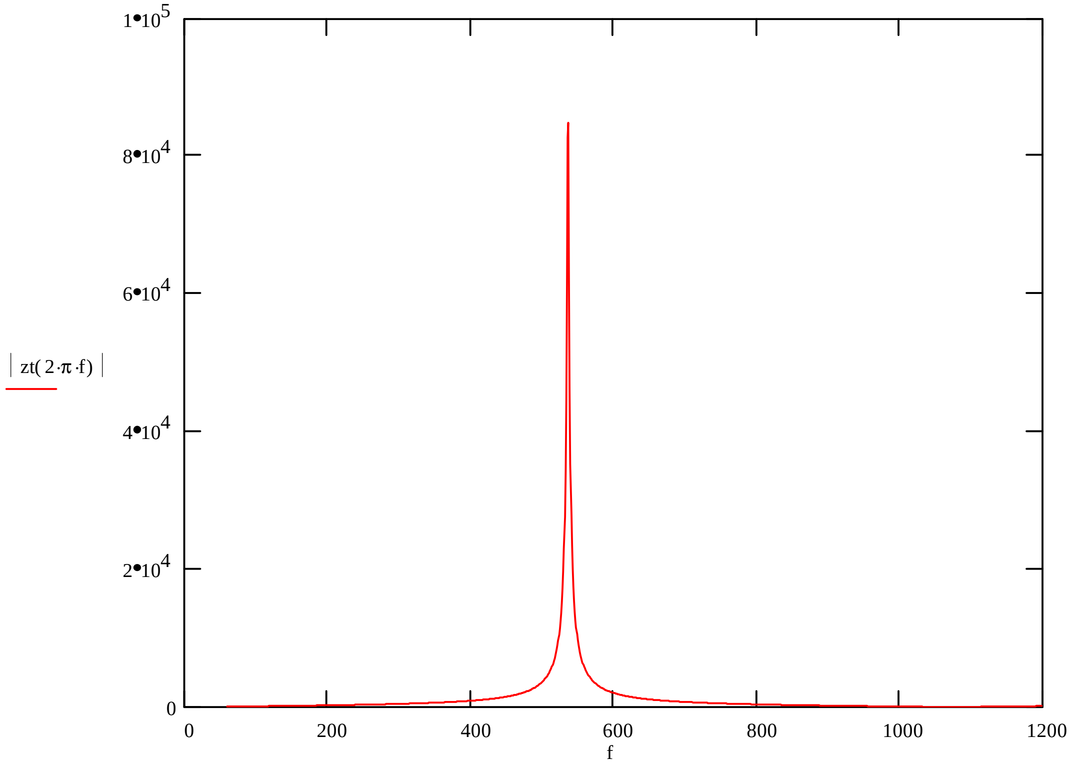
$$zsp(\omega) := 2.0\cdot\frac{1}{Yppos(\omega)} \qquad |zsp(\omega)| = 4409.598$$

$$zt(\omega) := \frac{Zspos(\omega)\cdot zsp(\omega)}{Zspos(\omega)+zsp(\omega)} \qquad |zt(\omega)| = 68.9962267 \quad arg(zt(\omega))\cdot\frac{180}{\pi} = 87.1077$$

$$f := 60, 120 .. 1200$$

f	$\lvert zt(2\pi\cdot f)\rvert$	$arg(zt(2\cdot\pi\cdot f))\cdot\frac{180}{\pi}$
60	68.99623	87.1077
120	142.34703	88.5054
180	225.93822	88.9438
240	328.79081	89.13176
300	468.02754	89.20154
360	683.04762	89.17565
420	1093.25431	88.9976
480	2322.5476	88.27375
540	35370.24135	-66.57714
600	2066.10149	-88.75704
660	1027.23169	-89.36947
720	652.13794	-89.54699
780	449.30328	-89.60162
840	315.58477	-89.58827
900	215.58083	-89.50266
960	133.51937	-89.27135
1020	60.90676	-88.44069
1080	7.99392	77.60038
1140	77.0139	88.60616
1200	151.237	89.18444

$f := 60, 61 .. 1200$



SuperHarm Benchmarking - SERIESFILTER Modeling Equations

Electrotek Concepts - 6/14/98, TEG

This document illustrates the equations that define the operation of the SERIESFILTER model.

SERIESFILTER Model Verification:

branch
name=zsrc
from= src
to=node
x=0.1

seriesfilter
name = filter
capbus = node
midbus = react1
indbus = GROUND
kv=7.9674
kva=1200
xratio = 20
harmonic = 4.7

$Xs := 0.1 \qquad Rs := 0 \qquad h := 4.7 \qquad f := 60$

$MVAr := 1.2 \qquad kV := \frac{13.8}{\sqrt{3}} \qquad kV = 7.967 \qquad ratio := 20$

Case 13a, 13b, & 13c:

$Ls := \frac{Xs}{2 \cdot \pi \cdot 60} \qquad Ls = 2.653 \cdot 10^{-4}$ at 60 Hz:

$Xc := \frac{kV^2 \cdot -1 \cdot i}{MVAr} \qquad Xc = -52.9i \qquad C := \frac{-1}{Xc \cdot 2 \cdot \pi \cdot f} \qquad C = -5.014 \cdot 10^{-5} i$

$Xf := \frac{-1 \cdot Xc}{h^2} \qquad Xf = 2.395i \qquad Rf := \frac{|Xf|}{ratio} \qquad Rf = 0.12$

$Xc(freq) := \frac{Xc}{\left(\frac{freq}{f}\right)} \qquad xf(freq) := \frac{freq}{f} \cdot Xf$

$Zf(freq) := Rf + (Xc(freq) + xf(freq)) \qquad Zf(60) = 0.12 - 50.505i$

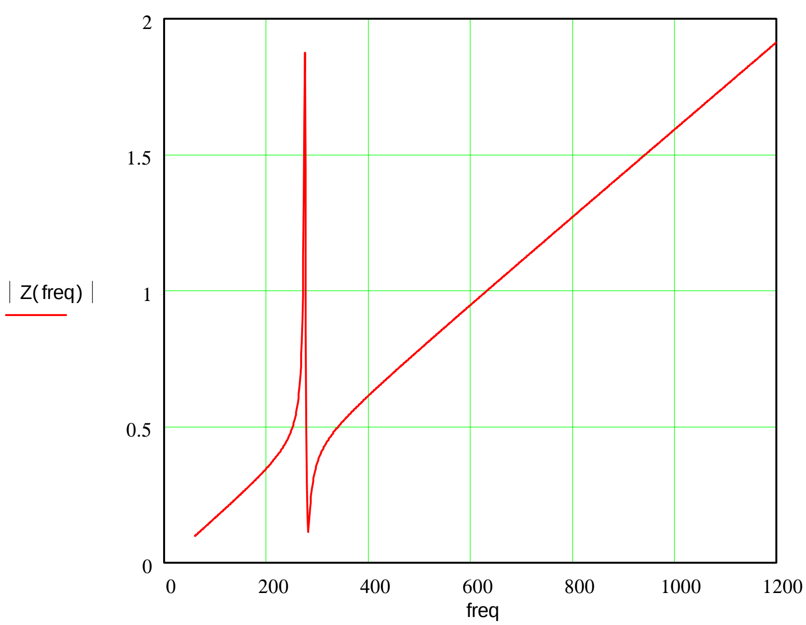
$Xs(freq) := j \cdot 2 \cdot \pi \cdot \frac{freq}{60} \cdot f \cdot Ls$

$Z(freq) := \frac{Zf(freq) \cdot Xs(freq)}{Zf(freq) + Xs(freq)}$

freq := 60, 120 .. 1200

freq	|Z(freq)|	$(\arg(Z(freq)))\cdot\frac{180}{\pi}$	|Zf(freq)|	$(\arg(Zf(freq)))\cdot\frac{180}{\pi}$
60	0.100198	90	50.505	-89.864
120	0.201864	89.997	21.661	-89.683
180	0.308867	89.981	10.45	-89.343
240	0.449228	89.768	3.648	-88.119
300	0.368606	88.708	1.399	85.09
360	0.541504	89.88	5.553	88.764
420	0.650543	89.947	9.207	89.255
480	0.752048	89.967	12.546	89.453
540	0.851134	89.976	15.675	89.562
600	0.949131	89.981	18.658	89.632
660	1.04654	89.985	21.533	89.681
720	1.143594	89.987	24.329	89.718
780	1.240415	89.988	27.063	89.746
840	1.337075	89.99	29.748	89.769
900	1.433619	89.991	32.395	89.788
960	1.530074	89.991	35.01	89.804
1020	1.626462	89.992	37.599	89.818
1080	1.722796	89.993	40.167	89.829
1140	1.819088	89.993	42.716	89.839
1200	1.915344	89.994	45.25	89.848

freq := 60, 61 .. 1200



SuperHarm Benchmarking - SWITCH Modeling Equations

Electrotek Concepts - 6/14/98, TEG

This document illustrates the equations that define the operation of the SWITCH model.

SWITCH Model Verification:

$Xs := 0.5$ $Rs := 0$ $Xl := 0.5$ $Rl := 0$ $f := 60$

Cases 14a:

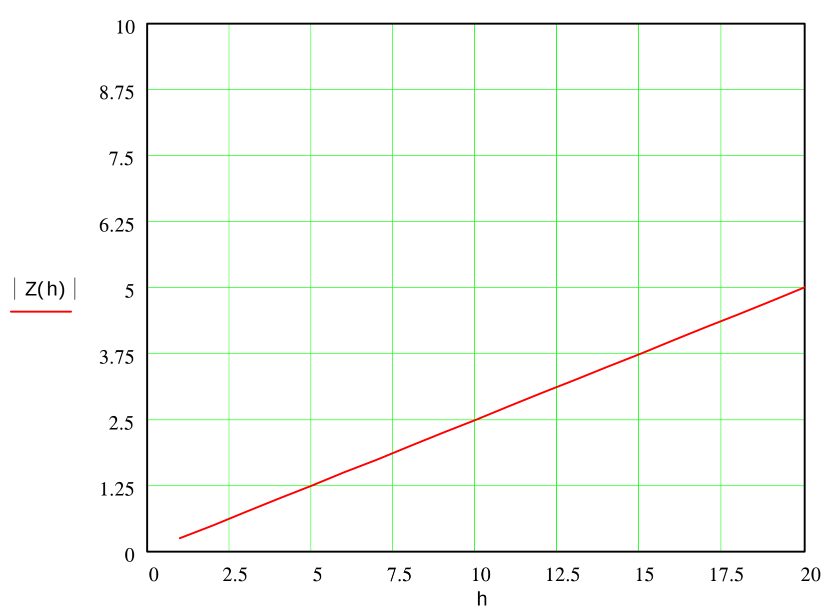
$Ls := \frac{Xs}{2\cdot\pi\cdot 60}$ $Ls = 1.326\cdot 10^{-3}$ at 60 Hz:

$Ll := \frac{Xl}{2\cdot\pi\cdot 60}$ $Ll = 1.326\cdot 10^{-3}$

$Xs(h) := j\cdot 2\cdot\pi\cdot h\cdot f\cdot Ls$ $Xl(h) := j\cdot 2\cdot\pi\cdot h\cdot f\cdot Ll$ $Z(h) := \frac{Xl(h)\cdot Xs(h)}{Xl(h) + Xs(h)}$

$h := 1 .. 20$

h	$\lvert Z(h)\rvert$	$(\arg(Z(h)))\cdot\frac{180}{\pi}$
1	0.25	90
2	0.5	90
3	0.75	90
4	1	90
5	1.25	90
6	1.5	90
7	1.75	90
8	2	90
9	2.25	90
10	2.5	90
11	2.75	90
12	3	90
13	3.25	90
14	3.5	90
15	3.75	90
16	4	90
17	4.25	90
18	4.5	90
19	4.75	90
20	5	90



SuperHarm Benchmarking - SWITCH Modeling Equations

Electrotek Concepts - 6/29/98, TEG

This document illustrates the equations that define the operation of the SWITCH model.

SWITCH Model Verification:

$Xs := 100000000$ $Rs := 100000000$ $Xl := 0.5$ $Rl := 0$ $f := 60$

[large impedance to simulate open switch]

Cases 14b:

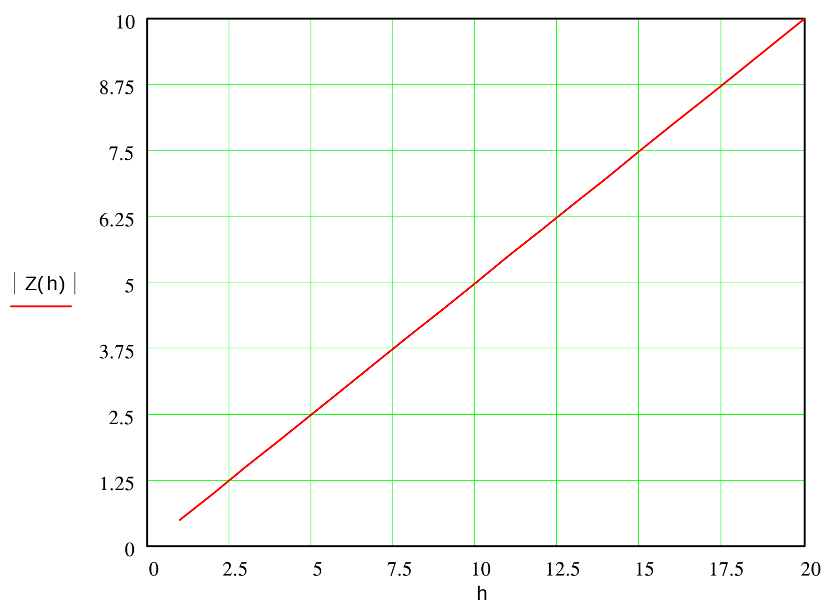
$Ls := \frac{Xs}{2 \cdot \pi \cdot 60}$ $Ls = 2.653 \cdot 10^{5}$ at 60 Hz:

$Ll := \frac{Xl}{2 \cdot \pi \cdot 60}$ $Ll = 1.326 \cdot 10^{-3}$

$Xs(h) := j \cdot 2 \cdot \pi \cdot h \cdot f \cdot Ls$ $Xl(h) := j \cdot 2 \cdot \pi \cdot h \cdot f \cdot Ll$ $Z(h) := \frac{Xl(h) \cdot Xs(h)}{Xl(h) + Xs(h)}$

$h := 1 .. 20$

h	$\lvert Z(h) \rvert$	$(\arg(Z(h))) \cdot \frac{180}{\pi}$
1	0.5	90
2	1	90
3	1.5	90
4	2	90
5	2.5	90
6	3	90
7	3.5	90
8	4	90
9	4.5	90
10	5	90
11	5.5	90
12	6	90
13	6.5	90
14	7	90
15	7.5	90
16	8	90
17	8.5	90
18	9	90
19	9.5	90
20	10	90



SuperHarm Benchmarking - TRANSFORMER Modeling Equations

Electrotek Concepts - 7/23/98, EWG/TEG

This document illustrates the equations that define the operation of a two winding transformer (TRANSFORMER). The input data for the calculation consists of the transformer short circuit test data, base mva, winding voltage ratings, transformer mva rating, and pu magnetizing impedance. This document uses Mathcad's units mechanism to ensure consistency and correctness of units.

TRANSFORMER Model Verification:

Cases 15a, 15b, & 15c:

```
transformer          name = xfmr1
            x.1  = n.1
            h.1  = n.2
            h.2 = ground
            mva  = 5
            %x.hx = 6
            kv.x =.480
            kv.h = 13.8
            xrconstant=yes
```

Input Data:

$f := 60 \quad \omega := 2 \cdot \pi \cdot f \qquad \omega = 376.99112$

$MVA_r := 5.0 \qquad MVA_b := 5.0$

$kv_H := 13.799249 \qquad kv_H = 13.8$

$kv_X := 0.480 \qquad kv_X = 0.48$

$X_{pu} := \frac{6.0}{100} \qquad X_{pu} = 0.06$

$R_{pu} := X_{pu} \cdot 0.04 \qquad R_{pu} = 0.0024$

$IMag_{pu} := 0.01$

Calculate the base impedances for the impedance data from the supplied base MVA:

$$ZbH := \frac{kv_H^{\,2}}{MVA_b} \qquad ZbH = 38.08385$$

$$ZbX := \frac{kv_X^{\,2}}{MVA_b} \qquad ZbX = 0.04608$$

Calculate the high side base admittance for the magnetizing branch from the transformer rated MVA:

$$Ymbase := \frac{MVA_r}{kv_H^2} \qquad Ymbase = 0.02626$$

Calculate the rated (base) current for the tranformer on the high side:

$$IbH := \frac{MVA_r}{kv_H} \qquad IbH = 0.36234$$

Calculate the high side magnetizing current:

$$IMag := IMag_{pu} \cdot IbH \qquad IMag = 3.62339 \cdot 10^{-3}$$

Calculate the high side magntizing admittance as a function of frequency
(note that Ipu == Ypu in the per unit system):

$$YMag_{pu} := IMag_{pu} \qquad YMag_{pu} = 0.01$$

$$Ym_{pu}(freq) := \frac{YMag_{pu}}{j} \cdot \frac{60}{freq} \qquad Ym_{pu}(f) = -0.01j$$

Create an equation for the pu leakage admittance as a function of frequency:

$$Y_{pu}(freq) := \frac{1}{R_{pu} + j \cdot X_{pu}} \cdot \frac{60}{freq} \qquad Y_{pu}(f) = 0.6656 - 16.64004j$$

Create an equation for the pu primative admittance matrix for the complete transformer as a function of frequency scaled to a single base MVA (1 MVA) to facilitate the conversion to engineering units:

$$S_{base} := 1$$

$$Yprim_{pu}(x) := \frac{1}{S_{base}} \cdot \begin{bmatrix} Y_{pu}(x) \cdot MVA_b + \frac{Ym_{pu}(x)}{2} \cdot MVA_r & -Y_{pu}(x) \cdot MVA_b \\ -Y_{pu}(x) \cdot MVA_b & Y_{pu}(x) \cdot MVA_b + \frac{Ym_{pu}(x)}{2} \cdot MVA_r \end{bmatrix}$$

$$Yprim_{pu}(f) = \begin{bmatrix} 3.32801 - 83.22521j & -3.32801 + 83.20021j \\ -3.32801 + 83.20021j & 3.32801 - 83.22521j \end{bmatrix}$$

Create an equation for the primative admittance matrix for the transformer in system units:

$$Y_{prim}(freq) := S_{base} \cdot \begin{bmatrix} \frac{Yprim_{pu}(freq)_{0,0}}{kv_H \cdot kv_H} & \frac{Yprim_{pu}(freq)_{0,1}}{kv_H \cdot kv_X} \\ \frac{Yprim_{pu}(freq)_{1,0}}{kv_X \cdot kv_H} & \frac{Yprim_{pu}(freq)_{1,1}}{kv_X \cdot kv_X} \end{bmatrix}$$

$$Y_{prim}(f) = \begin{bmatrix} 0.01748 - 0.43706j & -0.50244 + 12.5611j \\ -0.50244 + 12.5611j & 14.44448 - 361.22054j \end{bmatrix}$$

Calculate the impedance matrix as a function of frequency:

$$Z_{prim}(freq) := Y_{prim}(freq)^{(-1)}$$

$$Z_{prim}(f) = \begin{bmatrix} 0.02284 + 3.80896 \cdot 10^{3} j & -7.94598 \cdot 10^{-4} + 132.45292j \\ -7.94598 \cdot 10^{-4} + 132.45292j & 2.76397 \cdot 10^{-5} + 4.60869j \end{bmatrix}$$

Test the resulting admittance matrix by applying a voltage to the high side and calculating the resulting low side voltage.

$$VH := 7.96743 \cdot 1000$$

$$VX := \frac{-Y_{prim}(f)_{1,0} \cdot VH}{Y_{prim}(f)_{1,1}} \qquad VX = 277.05996 + 3.32372 \cdot 10^{-3} j$$

Now that we have calculated the terminal voltages, we can calculate the terminal currents and check for a match with the initial specified magneetizing current:

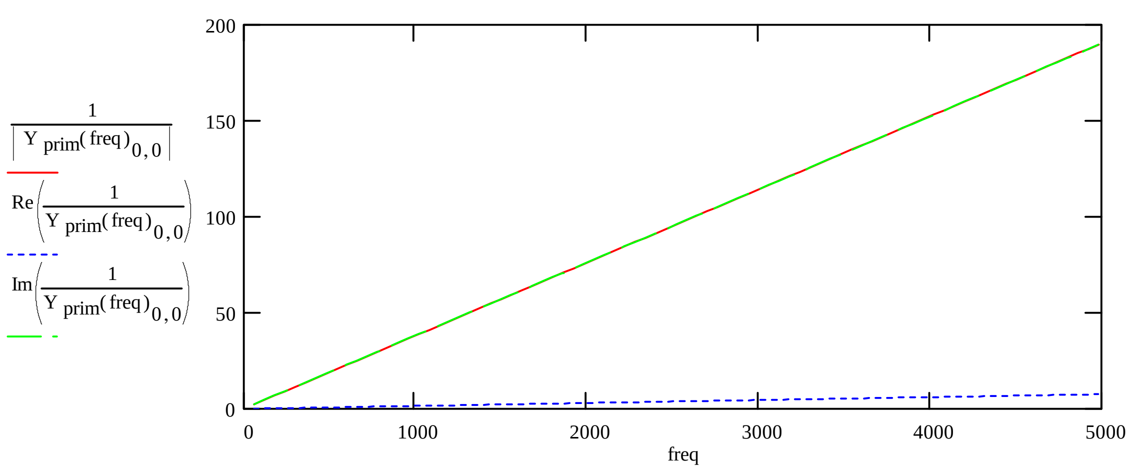
$$V := \begin{bmatrix} VH \\ VX \end{bmatrix} \qquad V = \begin{bmatrix} 7.96743 \cdot 10^{3} \\ 277.05996 + 3.32372 \cdot 10^{-3} j \end{bmatrix} \qquad |V_0| = 7967.43$$

$$|V_1| = 277.05996$$

$$I := Y_{prim}(f) \cdot V \qquad I = \begin{bmatrix} 1.25449 \cdot 10^{-5} - 2.09176j \\ 1.05027 \cdot 10^{-13} - 1.23396 \cdot 10^{-11} j \end{bmatrix} \qquad |I_0| = 2.09176$$

Plot the short circuit impedance of the transformer:

$$freq := 60, 120 .. 5000$$



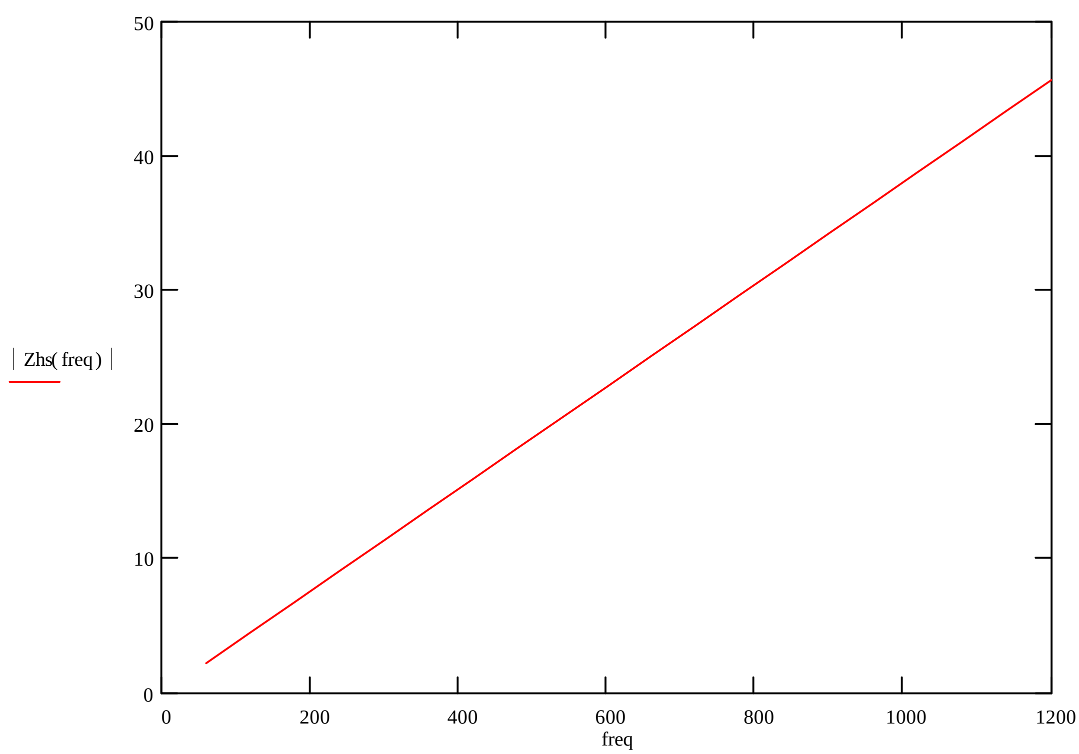
$$Zpu(f) := \frac{1}{Y_{pu}(f)} \qquad ZbX = 0.04608 \qquad ZbH = 38.08385$$

$$Zls(f) := Zpu(f) \cdot ZbX$$

$$Y_{pu}(f) = 0.6656 - 16.64004j \qquad Zpu(60) = 2.4 \cdot 10^{-3} + 0.06j \qquad Zhs(f) := (Zpu(f)) \cdot ZbH$$

$freq := 60, 120 .. 1200$

$freq$	$\lvert Y_{pu}(freq) \rvert$	$\lvert Zls(freq) \rvert$	$\lvert Zhs(freq) \rvert$	$arg(Zhs(freq)) \cdot \frac{180}{\pi}$	$\lvert Zpu(freq) \rvert$
60	16.65335	$2.76701 \cdot 10^{-3}$	2.2869	87.70939	0.06005
120	8.32667	$5.53402 \cdot 10^{-3}$	4.5737	87.70939	0.1201
180	5.55112	$8.30103 \cdot 10^{-3}$	6.8606	87.70939	0.18014
240	4.16334	0.01107	9.1474	87.70939	0.24019
300	3.33067	0.01384	11.4343	87.70939	0.30024
360	2.77556	0.0166	13.7212	87.70939	0.36029
420	2.37905	0.01937	16.008	87.70939	0.42034
480	2.08167	0.02214	18.2949	87.70939	0.48038
540	1.85037	0.0249	20.5817	87.70939	0.54043
600	1.66533	0.02767	22.8686	87.70939	0.60048
660	1.51394	0.03044	25.1554	87.70939	0.66053
720	1.38778	0.0332	27.4423	87.70939	0.72058
780	1.28103	0.03597	29.7292	87.70939	0.78062
840	1.18952	0.03874	32.016	87.70939	0.84067
900	1.11022	0.04151	34.3029	87.70939	0.90072
960	1.04083	0.04427	36.5897	87.70939	0.96077
1020	0.97961	0.04704	38.8766	87.70939	1.02082
1080	0.92519	0.04981	41.1635	87.70939	1.08086
1140	0.87649	0.05257	43.4503	87.70939	1.14091
1200	0.83267	0.05534	45.7372	87.70939	1.20096



SuperHarm Benchmarking - TRANSFORMER Modeling Equations

Electrotek Concepts - 7/23/98, EWG/TEG

This document illustrates the equations that define the operation of a three winding transformer (TRANSFORMER). The input data for the calculation consists of the transformer short circuit test data, base mva's, winding voltage ratings, h-x mva rating, and pu magnetizing impedance. This document uses Mathcad's units mechanism to ensure consistency and correctness of units.

TRANSFORMER Model Verification:

Cases 15e, & 15f:

```
transformer name=trans
h = wye         x = wye              t = delta
x.a = src.a     x.b = src.b   x.c = src.c
h.a = n2.a      h.b = n2.b    h.c = n2.c
t.a = nt.a      t.b = nt.b        t.c = nt.c
mva  = 5  kv.x =.480  kv.h = 13.8  kv.t = 0.277
mvab.hx = 30.0  mvab.ht = 30.0  mvab.xt = 30.0
%x.hx = 36  %r.hx = 1.44
%x.ht = 36  %r.ht = 1.44
%x.xt = 36  %r.xt = 1.44
xrconstant=yes
```

Input Data:

$MVA_r := 5.0$ $IMag_{pu} := 0.01$

$kv_h := 13.8$ $Rhx_{pu} := 0.01440$ $Xhx_{pu} := 0.3600$

$kv_x := 0.480$ $Rht_{pu} := 0.01440$ $Xht_{pu} := 0.3600$

$kv_t := 0.277$ $Rxt_{pu} := 0.01440$ $Xxt_{pu} := 0.3600$

$MVA_{hx} := 30$ $MVA_{ht} := 30$ $MVA_{xt} := 30$

$f := 60$ $h := 1$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 376.99112$

Convert the short circuit reactances to a common base MVA (1 MVA):

$S_{base} := 1$

$$Zhx_{pu} := \left(Rhx_{pu} + j \cdot Xhx_{pu}\right) \cdot \frac{S_{base}}{MVA_{hx}} \qquad Zhx_{pu} = 4.8 \cdot 10^{-4} + 0.012j$$

$$Zht_{pu} := \left(Rht_{pu} + j \cdot Xht_{pu}\right) \cdot \frac{S_{base}}{MVA_{ht}} \qquad Zht_{pu} = 4.8 \cdot 10^{-4} + 0.012j$$

$$Zxt_{pu} := \left(Rxt_{pu} + j \cdot Xxt_{pu}\right) \cdot \frac{S_{base}}{MVA_{xt}} \qquad Zxt_{pu} = 4.8 \cdot 10^{-4} + 0.012j$$

Calculate the star equivalent circuit from the short circuit test data:

$Zh_{pu} := \frac{1}{2} \cdot (Zhx_{pu} + Zht_{pu} - Zxt_{pu})$ $Zh_{pu} = 2.4 \cdot 10^{-4} + 6 \cdot 10^{-3} j$ $|Zh_{pu}| = 0.0060048$

$Zx_{pu} := \frac{1}{2} \cdot (Zxt_{pu} + Zhx_{pu} - Zht_{pu})$ $Zx_{pu} = 2.4 \cdot 10^{-4} + 6 \cdot 10^{-3} j$ $|Zx_{pu}| = 0.0060048$

$Zt_{pu} := \frac{1}{2} \cdot (Zht_{pu} + Zxt_{pu} - Zhx_{pu})$ $Zt_{pu} = 2.4 \cdot 10^{-4} + 6 \cdot 10^{-3} j$

Convert the start circuit into an equivalent mesh circuit as a function of frequency while assuming that the resistance is frequency dependent and maintains a constant X/R ratio:

$D := Zh_{pu} \cdot Zx_{pu} + Zx_{pu} \cdot Zt_{pu} + Zt_{pu} \cdot Zh_{pu}$

$Zhx_{pu}(freq) := \frac{D}{Zt_{pu}} \cdot \frac{freq}{60}$ $Zhx_{pu}(f) = 7.2 \cdot 10^{-4} + 0.018j$ $|Zhx_{pu}(f)| = 0.01801$

$Zht_{pu}(freq) := \frac{D}{Zx_{pu}} \cdot \frac{freq}{60}$ $Zht_{pu}(f) = 7.2 \cdot 10^{-4} + 0.018j$

$Zxt_{pu}(freq) := \frac{D}{Zh_{pu}} \cdot \frac{freq}{60}$ $Zxt_{pu}(f) = 7.2 \cdot 10^{-4} + 0.018j$

Calculate frequency dependent formulas for winding admittance:

$Yhx_{pu}(freq) := \frac{1}{Zhx_{pu}(freq)}$ $Yhx_{pu}(f) = 2.21867 - 55.46681j$

$Yht_{pu}(freq) := \frac{1}{Zht_{pu}(freq)}$ $Yht_{pu}(f) = 2.21867 - 55.46681j$

$Yxt_{pu}(freq) := \frac{1}{Zxt_{pu}(freq)}$ $Yxt_{pu}(f) = 2.21867 - 55.46681j$

Calculate the high side magnetizing current:

$IMag := IMag_{pu} \cdot \frac{MVA_r}{kv_h}$ $IMag = 3.62319 \cdot 10^{-3}$

Calculate the high side magntizing admittance (note that Ipu == Ypu in the per unit system) as a function of frequency on a 1 MVA base:

$YMag_{pu} := IMag_{pu}$

$Ym_{pu}(freq) := \frac{YMag_{pu}}{j} \cdot \frac{60}{freq} \cdot \frac{MVA_r}{S_{base}}$ $Ym_{pu}(f) = -0.05j$

Create an equation for the pu primative admittance matrix for the complete transformer as a function of frequency:

$$Dh(x) := Yhx_{pu}(x) + Yht_{pu}(x) + \frac{Ym_{pu}(x)}{3}$$

$$Dx(x) := Yhx_{pu}(x) + Yxt_{pu}(x) + \frac{Ym_{pu}(x)}{3}$$

$$Dt(x) := Yht_{pu}(x) + Yxt_{pu}(x) + \frac{Ym_{pu}(x)}{3}$$

$$Yprim_{pu}(x) := \begin{bmatrix} Dh(x) & -Yhx_{pu}(x) & -Yht_{pu}(x) \\ -Yhx_{pu}(x) & Dx(x) & -Yxt_{pu}(x) \\ -Yht_{pu}(x) & -Yxt_{pu}(x) & Dt(x) \end{bmatrix}$$

$$Yprim_{pu}(f) = \begin{bmatrix} 4.43734 - 110.95028j & -2.21867 + 55.46681j & -2.21867 + 55.46681j \\ -2.21867 + 55.46681j & 4.43734 - 110.95028j & -2.21867 + 55.46681j \\ -2.21867 + 55.46681j & -2.21867 + 55.46681j & 4.43734 - 110.95028j \end{bmatrix}$$

Create an equation for the primative admittance matrix for the transformer in system units:

$$Y_{prim}(freq) := S_{base} \cdot \begin{bmatrix} \frac{Yprim_{pu}(freq)_{0,0}}{kv_h \cdot kv_h} & \frac{Yprim_{pu}(freq)_{0,1}}{kv_h \cdot kv_x} & \frac{Yprim_{pu}(freq)_{0,2}}{kv_h \cdot kv_t} \\ \frac{Yprim_{pu}(freq)_{1,0}}{kv_x \cdot kv_h} & \frac{Yprim_{pu}(freq)_{1,1}}{kv_x \cdot kv_x} & \frac{Yprim_{pu}(freq)_{1,2}}{kv_x \cdot kv_t} \\ \frac{Yprim_{pu}(freq)_{2,0}}{kv_t \cdot kv_h} & \frac{Yprim_{pu}(freq)_{2,1}}{kv_t \cdot kv_x} & \frac{Yprim_{pu}(freq)_{2,2}}{kv_t \cdot kv_t} \end{bmatrix}$$

$$Y_{prim}(f) = \begin{bmatrix} 0.0233 - 0.5826j & -0.33494 + 8.37361j & -0.58041 + 14.51023j \\ -0.33494 + 8.37361j & 19.25931 - 481.55505j & -16.68677 + 417.16914j \\ -0.58041 + 14.51023j & -16.68677 + 417.16914j & 57.83139 - 1.446 \cdot 10^{3}j \end{bmatrix}$$

Calculate the impedance matrix as a function of frequency:

$$Z_{prim}(freq) := Y_{prim}(freq)^{(-1)}$$

$$Z_{prim}(f) = \begin{bmatrix} 0.03046 + 3.80956 \cdot 10^{3}j & -5.29814 \cdot 10^{-4} + 132.46675j & -3.05747 \cdot 10^{-4} + 76.44436j \\ -5.29814 \cdot 10^{-4} + 132.46675j & 3.68566 \cdot 10^{-5} + 4.60892j & -1.06347 \cdot 10^{-5} + 2.65893j \\ -3.05747 \cdot 10^{-4} + 76.44436j & -1.06347 \cdot 10^{-5} + 2.65893j & 1.22742 \cdot 10^{-5} + 1.53489j \end{bmatrix}$$

Test the resulting admittance matrix by applying a voltage to the high side and calculating the resulting low side voltage.

$V_h := 13800$

$Y_{nn} := \begin{bmatrix} Y_{prim}(f)_{1,1} & Y_{prim}(f)_{1,2} \\ Y_{prim}(f)_{2,1} & Y_{prim}(f)_{2,2} \end{bmatrix}$ $\qquad Y_{nv} := \begin{bmatrix} Y_{prim}(f)_{1,0} \\ Y_{prim}(f)_{2,0} \end{bmatrix}$

$B := -Y_{nv} \cdot V_h$

$V_n := Y_{nn}^{-1} \cdot B$ $\qquad V_n = \begin{bmatrix} 479.85604 + 5.75655 \cdot 10^{-3} j \\ 276.91692 + 3.32201 \cdot 10^{-3} j \end{bmatrix}$

Now that we have calculated the terminal voltages, we can calculate the terminal currents and check for a match with the initial specified magnetizing current:

$V := \begin{bmatrix} V_h \\ V_{n_0} \\ V_{n_1} \end{bmatrix}$ $\qquad V = \begin{bmatrix} 13800 \\ 479.85604 + 0.00576j \\ 276.91692 + 0.00332j \end{bmatrix}$

$I := Y_{prim}(f) \cdot V$ $\qquad I = \begin{bmatrix} 2.89681 \cdot 10^{-5} - 3.62246j \\ 2.00817 \cdot 10^{-12} - 1.42286 \cdot 10^{-11} j \\ -2.30704 \cdot 10^{-12} + 4.26304 \cdot 10^{-11} j \end{bmatrix}$

$|V_0| = 1.38 \cdot 10^4$

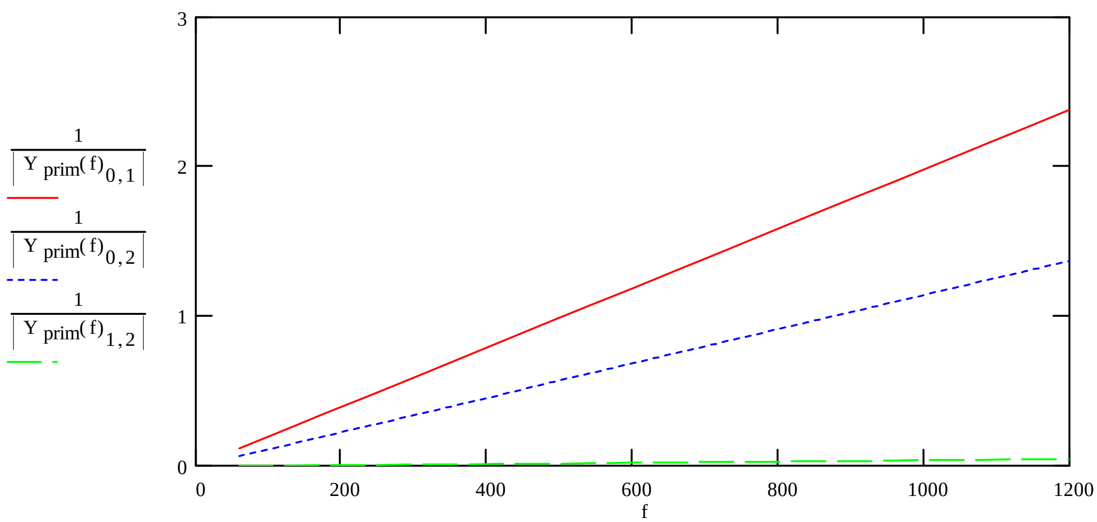
$|V_1| = 479.85604$

$|V_2| = 276.91692$

$|I_0| = 3.62246$

Plot the short circuit impedance of the transformer from h to x, h to t, and x to t:

$f := 60, 120 .. 1200$



$|Zhx_{pu}(60)| = 0.01801$

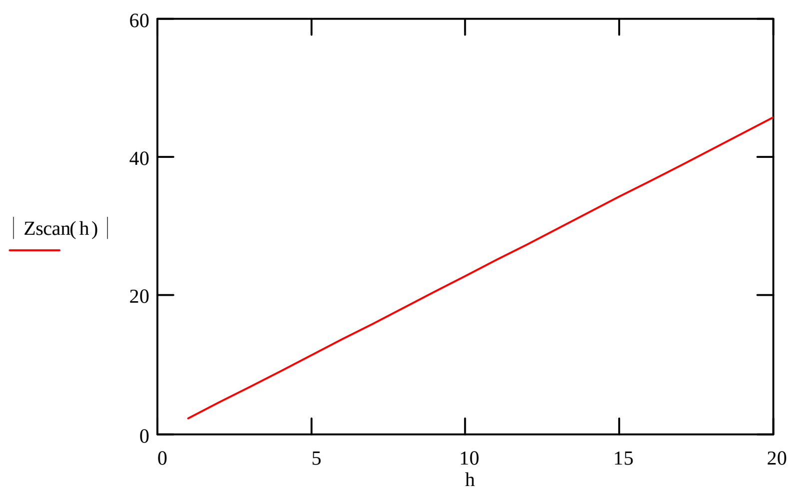
$ZbH := \frac{(kv_h)^2}{MVA_r}$ $\quad ZbH = 38.088$

$Zscan(h) := ZbH \cdot \frac{(MVA_r)}{MVA_{hx}} \cdot (h \cdot Rhx_{pu} + i \cdot h \cdot Xhx_{pu})$ $\quad Zscan(1) = 0.09141 + 2.28528j$

$|Zscan(1)| = 2.28711$

$h := 1, 2 .. 20$ $\quad arg(Zscan(1)) \cdot \frac{180}{\pi} = 87.70939$

h	Zscan(h)	|Zscan(h)|	$arg(Zscan(h)) \cdot \frac{180}{\pi}$
1	0.09141 + 2.28528j	2.28711	87.70939
2	0.18282 + 4.57056j	4.57421	87.70939
3	0.27423 + 6.85584j	6.86132	87.70939
4	0.36564 + 9.14112j	9.14843	87.70939
5	0.45706 + 11.4264j	11.43554	87.70939
6	0.54847 + 13.71168j	13.72264	87.70939
7	0.63988 + 15.99696j	16.00975	87.70939
8	0.73129 + 18.28224j	18.29686	87.70939
9	0.8227 + 20.56752j	20.58397	87.70939
10	0.91411 + 22.8528j	22.87107	87.70939
11	1.00552 + 25.13808j	25.15818	87.70939
12	1.09693 + 27.42336j	27.44529	87.70939
13	1.18835 + 29.70864j	29.7324	87.70939
14	1.27976 + 31.99392j	32.0195	87.70939
15	1.37117 + 34.2792j	34.30661	87.70939
16	1.46258 + 36.56448j	36.59372	87.70939
17	1.55399 + 38.84976j	38.88083	87.70939
18	1.6454 + 41.13504j	41.16793	87.70939
19	1.73681 + 43.42032j	43.45504	87.70939
20	1.82822 + 45.7056j	45.74215	87.70939



SuperHarm Benchmarking - ZYCMATRIX Modeling Equations

Electrotek Concepts - 6/19/98, TEG

This document illustrates the equations that define the operation of the ZYCMATRIX model.

ZYCMATRIX Model Verification:

Case 17a:

```
ZYCMATRIX NAME=ZMATRX LENGTH=6
ZMULT=3.0 YCMULT=@"1 6 /"
          FROM={SRCA, SRCB, SRCC}
          TO={BUS1A, BUS1B, BUS1C}

!  Ohms/Mile

ZRMATRIX={
         {0.2493, 0.0520, 0.0520},
         {0.0520, 0.2493, 0.0520},
         {0.0520, 0.0520, 0.2493}
         }
ZXMATRIX={
         {0.3987, 0.1443, 0.1443},
         {0.1443, 0.3987, 0.1443},
         {0.1443, 0.1443, 0.3987}
         }

!  Siemens/Mile

YCMATRIX= {
          {5.655e-6, 0, 0},
          {0, 5.655e-6, 0},
          {0, 0, 5.655e-6}
          }
```

$3 \cdot 0.2493 \cdot 6 = 4.4874$ $\quad 3 \cdot 0.052 \cdot 6 = 0.936$

$3 \cdot 0.3987 \cdot 6 = 7.1766$ $\quad 3 \cdot 0.1443 \cdot 6 = 2.5974$

$\left(\frac{1}{6} \cdot 5.655 \cdot 10^{-6} \cdot 6\right)^{-1} = 176834.659593$

$$Zmatrix := \begin{bmatrix} 4.4874 + i \cdot 7.1766 & 0.9360 + i \cdot 2.5974 & 0.9360 + i \cdot 2.5974 \\ 0.9360 + i \cdot 2.5974 & 4.4874 + i \cdot 7.1766 & 0.9360 + i \cdot 2.5974 \\ 0.9360 + i \cdot 2.5974 & 0.9360 + i \cdot 2.5974 & 4.4874 + i \cdot 7.1766 \end{bmatrix}$$

$$Zseq := \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 + i \cdot 0.866025 & -0.5 - i \cdot 0.866025 \\ 1 & -0.5 - i \cdot 0.866025 & -0.5 + i \cdot 0.866025 \end{bmatrix} \cdot Zmatrix \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 - i \cdot 0.866025 & -0.5 + i \cdot 0.866025 \\ 1 & -0.5 + i \cdot 0.866025 & -0.5 - i \cdot 0.866025 \end{bmatrix}$$

$$Zseq = \begin{bmatrix} 6.359 + 12.371i & 0 & 0 \\ 0 & 3.551 + 4.579i & 1.656 \cdot 10^{-6} + 2.135 \cdot 10^{-6} i \\ 0 & 1.656 \cdot 10^{-6} + 2.135 \cdot 10^{-6} i & 3.551 + 4.579i \end{bmatrix}$$

$$Zseq1 := \begin{bmatrix} 6.366 + 12.3661i & 0 & 0 \\ 0 & 3.552 + 4.578i & 0 \\ 0 & 0 & 3.552 + 4.578i \end{bmatrix} \qquad \frac{Zseq1_{1,1}}{6} = 0.592 + 0.763i$$

$$Zmatrix1 := \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & (-0.5 + i \cdot 0.866) & (-0.5 - i \cdot 0.866) \\ 1 & (-0.5 - i \cdot 0.866) & (-0.5 + i \cdot 0.866) \end{bmatrix} \cdot Zseq1 \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & (-0.5 - i \cdot 0.866) & (-0.5 + i \cdot 0.866) \\ 1 & (-0.5 + i \cdot 0.866) & (-0.5 - i \cdot 0.866) \end{bmatrix}$$

$$Zmatrix1 = \begin{bmatrix} 4.49 + 7.174i & 0.938 + 2.596i & 0.938 + 2.596i \\ 0.938 + 2.596i & 4.49 + 7.174i & 0.938 + 2.596i \\ 0.938 + 2.596i & 0.938 + 2.596i & 4.49 + 7.174i \end{bmatrix}$$

$$Zpseq := Zseq_{1,1} \qquad Zpseq = 3.551 + 4.579i \qquad |Zpseq| = 5.795 \qquad \arg(Zpseq) \cdot \frac{180}{\pi} = 52.205$$

$$Zcap := \begin{bmatrix} 176834.659593 & 0 & 0 \\ 0 & 176834.659593 & 0 \\ 0 & 0 & 176834.659593 \end{bmatrix}$$

$$ZCmatrix1 := \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & (-0.5 + i \cdot 0.866025) & (-0.5 - i \cdot 0.866025) \\ 1 & (-0.5 - i \cdot 0.866025) & (-0.5 + i \cdot 0.866025) \end{bmatrix} \cdot Zcap \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & (-0.5 - i \cdot 0.866025) & (-0.5 + i \cdot 0.866025) \\ 1 & (-0.5 + i \cdot 0.866025) & (-0.5 - i \cdot 0.866025) \end{bmatrix}$$

$$ZCmatrix1 = \begin{bmatrix} 176834.66 & 0 & 0 \\ 0 & 176834.577 & 0.082 \\ 0 & 0.082 & 176834.577 \end{bmatrix}$$

$$YCmatrix1 := ZCmatrix1^{-1}$$

$$Xc := ZCmatrix1_{1,1}$$

$$Xc = 176834.577$$

$$YCmatrix1 = \begin{bmatrix} 5.655 \cdot 10^{-6} & 0 & 0 \\ 0 & 5.655 \cdot 10^{-6} & -2.637 \cdot 10^{-12} \\ 0 & -2.637 \cdot 10^{-12} & 5.655 \cdot 10^{-6} \end{bmatrix}$$

$Zpseq := Zseq_{1,1}$ $Zpseq = 3.551 + 4.579i$ $|Zpseq| = 5.795$ $arg(Zpseq) \cdot \frac{180}{\pi} = 52.205$

$Rpseq := Re(Zpseq)$ $Xpseq := Im(Zpseq)$ $Rpseq = 3.551$ $Xpseq = 4.579$

$h := 1$

$Zhpseq(h) := Rpseq + i \cdot h \cdot Xpseq$ $|Zhpseq(1)| = 5.795$ $arg(Zhpseq(1)) \cdot \frac{180}{\pi} = 52.205$

$h := 1, 2 .. 5$

h	$\lvert Zhpseq(h) \rvert$	$arg(Zhpseq(h)) \cdot \frac{180}{\pi}$
1	5.795	52.205
2	9.823	68.805
3	14.189	75.505
4	18.658	79.027
5	23.17	81.183

$Xc = 1.768 \cdot 10^5$

$Xc(h) := \frac{-i \cdot Xc}{h}$

h	$\lvert Xc(h) \rvert$	$arg(Xc(h)) \cdot \frac{180}{\pi}$
1	176834.58	-90
2	88417.29	-90
3	58944.86	-90
4	44208.64	-90
5	35366.92	-90

$Zt(h) := \frac{(Zhpseq(h) \cdot Xc(h))}{Zhpseq(h) + Xc(h)}$

h	$\lvert Zt(h) \rvert$	$arg(Zt(h)) \cdot \frac{180}{\pi}$
1	5.8	52.203
2	9.82	68.803
3	14.19	75.502
4	18.67	79.023
5	23.18	81.177

$h := 1, 2 .. 20$

h	$\lvert Zt(h) \rvert$	$\arg(Zt(h)) \cdot \frac{180}{\pi}$	$\lvert\lvert Zhpseq(h) \rvert\rvert$	$\arg(Zhpseq(h)) \cdot \frac{180}{\pi}$
1	5.8	52.203	5.795	52.205
2	9.82	68.803	9.823	68.805
3	14.19	75.502	14.189	75.505
4	18.67	79.023	18.658	79.027
5	23.18	81.177	23.17	81.183
6	27.73	82.628	27.704	82.635
7	32.29	83.67	32.251	83.678
8	36.87	84.454	36.805	84.463
9	41.45	85.064	41.366	85.075
10	46.05	85.554	45.929	85.565
11	50.65	85.954	50.496	85.967
12	55.27	86.288	55.065	86.302
13	59.9	86.571	59.635	86.586
14	64.53	86.813	64.207	86.829
15	69.18	87.023	68.78	87.04
16	73.84	87.206	73.353	87.225
17	78.51	87.368	77.927	87.388
18	83.2	87.512	82.502	87.533
19	87.9	87.641	87.077	87.663
20	92.61	87.756	91.653	87.779

$h := 1, 1.1 .. 1000$